Frequency Modulation: Basic Principles

- To generate an angle modulated signal, the amplitude of the modulated carrier is held constant, while either the phase or the time derivative of the phase is varied linearly with the message signal m(t).
- $\mathbf{c}(\boldsymbol{t}) = \boldsymbol{A_c} \cos(2\pi \boldsymbol{f_c t})$; unmodulated carrier
- The expression for an angle modulated signal is
- $\boldsymbol{s(t) = A_c \cos\big(2\pi f_c t + \theta(t)\big)}$,
- $\theta(t)$: A phase difference that contains the information message.
- f_c is the carrier frequency in Hz.
- The instantaneous frequency of s(t) is :
- $f_i(t) = \frac{1}{2\pi}\frac{d}{dt}\big(2\pi f_c t + \theta(t)\big)$
- $\boldsymbol{f_i(t) = f_c + \frac{1}{2\pi}\frac{d\theta(t)}{dt}}$

- Since the information is contained in the phase, this type of modulation is less susceptible to AWGN and interference from electrical equipment and the atmosphere.
- For **phase modulation**, the phase $\theta(t)$ is directly proportional to the modulating signal

$$\boldsymbol{\theta(t) = k_p m(t),}$$

- k_p is the phase sensitivity measured in rad/volt.
- The time domain representation of a phase modulated signal is
- $\boldsymbol{s(t)_{PM} = A_c \cos\Big(2\pi f_c t + k_p m(t)\Big)}$.
- The instantaneous frequency of $\boldsymbol{s(t)_{PM}}$ is :
- $\boldsymbol{f_i(t) = f_c + \frac{k_p}{2\pi}\frac{dm(t)}{dt}}$

Frequency Modulation

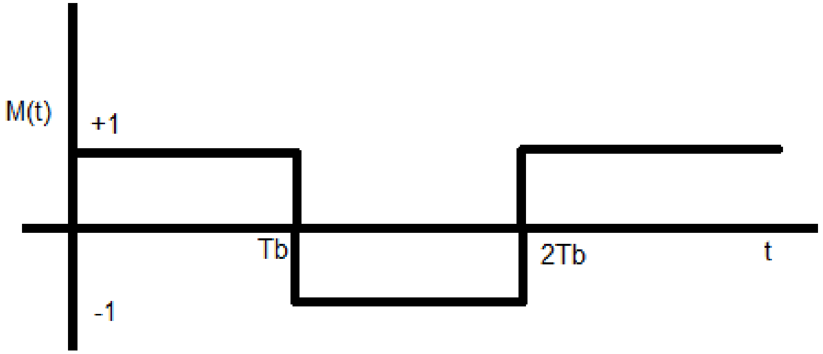
- An angle modulated signal is
- $s(t) = A_c \cos\left(2\pi f_c t + \theta(t)\right)$;
- The instantaneous frequency of s(t) is :
- $f_i(t) = f_c + \frac{1}{2\pi}\frac{d\theta(t)}{dt}$
- For **frequency modulation**, the frequency deviation of the carrier is proportional to the modulating signal:
- $\frac{1}{2\pi}\frac{d\theta(t)}{dt} = k_f m(t)$
- Integrating both sides, we get
- $\boldsymbol{\theta(t) = 2\pi k_f \int_0^t m(\alpha)d\alpha}$
- The instantaneous frequency beomes
- $\boldsymbol{f_i(t) = f_c + k_f m(t)}$;
- $f_i(t) - f_c = k_f m(t)$.
- The peak frequency deviation is
- $\Delta f = \max\{k_f m(t)\}$.
- The time domain representation of a frequency modulated signal is
- $\boldsymbol{s(t)_{FM} = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\alpha)d\alpha\right)}$.
- Where $\boldsymbol{\theta(t) = 2\pi k_f \int_0^t m(\alpha)d\alpha}$
- The average power in $s(t)$, for frequency modulation (FM) or phase modulation (PM) is: $\boldsymbol{p_{av} = \frac{(A_c)^2}{2} = constant}$.

Example: Binary Frequency Shift Keying

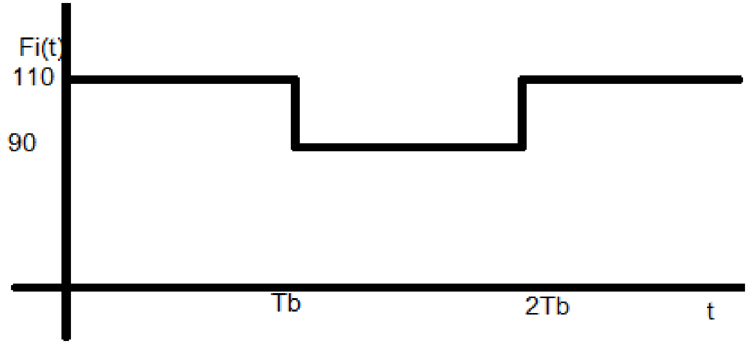
- The periodic square signal m(t), shown below, frequency modulates the carrier $c(t) = A_c \cos(2\pi 100 t)$ to produce the FM signal

  $s(t) = A_c \cos\left(2\pi 100 t + 2\pi k_f \int m(\alpha) d\alpha\right)$ where $k_f = 10 Hz/V$.

- Find and plot the instantaneous frequency $f_i(t)$.
- Find and sketch the time domain expression for $s(t)$.



- **Solution**: The instantaneous frequency is
- $f_i = f_c + k_f m(t)$
- f_i = 100+10 = 110 when $m(t) = +1$ $(0 < t \leq T_b)$
- f_i = 100 -10 = 90Hz when $m(t) = -1$ $(T_b \leq t \leq 2T_b)$



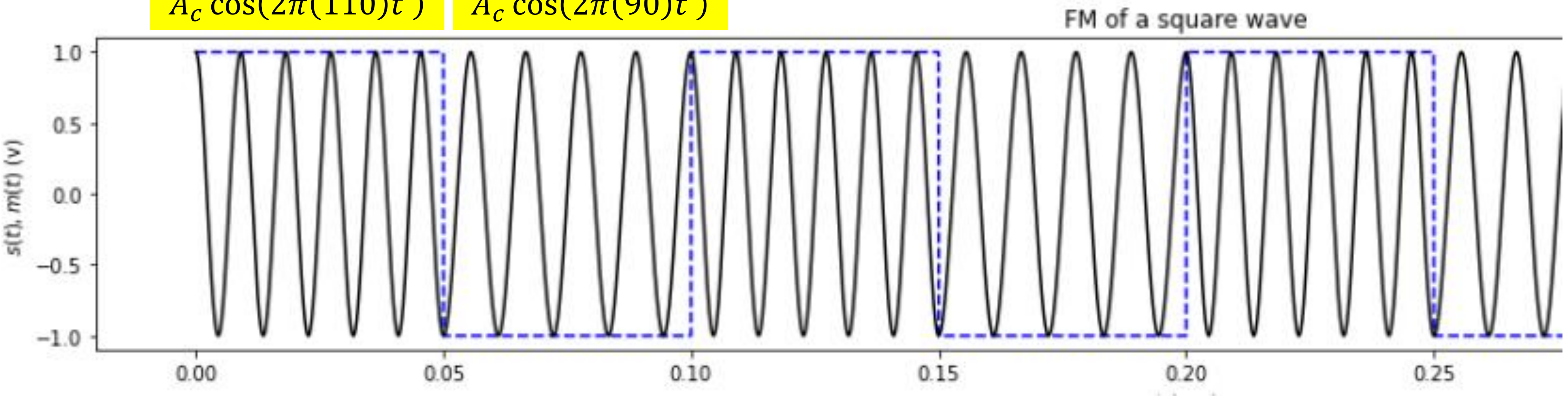
- **Remark:** In digital transmission, we will see that a binary (1) may be represent by a signal of frequency f_1 for $0 \leq t \leq T_b$ and a binary (0) by a signal of frequency f_2 for $0 \leq t \leq T_b$ (This type of digital modulation is called FSK)

Example: Binary Frequency Shift Keying

Solution: The instantaneous frequency is $\boldsymbol{f_i = f_c + k_f m(t)}$

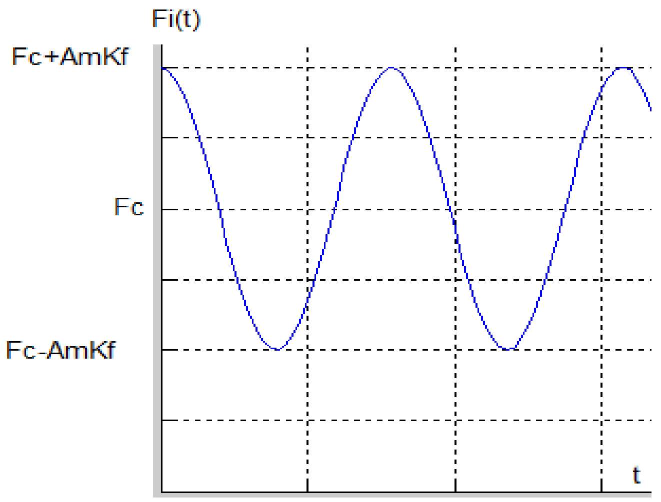
- $f_i =$ 100+10 = 110 when $m(t) = +1$ ($0 < t \leq T_b$)
- $f_i =$ 100 -10 = 90Hz when $m(t) = -1$ ($T_b \leq t \leq 2T_b$)
- The instantaneous frequency hops between the two values 110 Hz and 90 Hz as shown.
- Depending on the input binary digit, $s(t)$ may take any one of the following expressions
- $s(t) = A_c \cos(2\pi(110)t\ ),\ \ when\ m(t) = +1$
- $s(t) = A_c \cos(2\pi(90)t\ ),\ when\ m(t) = -1$

$A_c \cos(2\pi(110)t\ )$ $A_c \cos(2\pi(90)t\ )$



Single Tone Frequency Modulation

- Assume that the message $m(t) = A_m \cos \omega_m t$.
- The instantaneous frequency is: $\boldsymbol{f_i = f_c + k_f m(t) = f_c + A_m k_f \cos 2\pi f_m t.}$
- This frequency is plotted in the figure.
- The peak frequency deviation (from the un-modulated carrier) is : $\Delta f = k_f A_m$.
- The FM signal is: $\boldsymbol{s(t)_{FM} = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\alpha)d\alpha\right)}$
- $s(t)_{FM} = A_c\, cos\left(2\pi f_c t + 2\pi k_f \int_0^t A_m \cos \omega_m \alpha \, d\alpha\right)$
- $s(t) = A_c \cos\ (2\pi f_c t + \frac{k_f A_m}{f_m} \sin 2\pi f_m t)$.
- $\boldsymbol{s(t) = A_c \cos\ (2\pi f_c t + \beta \sin 2\pi f_m t)}$.
- Where β is the **FM modulation index**, defined as
- $\beta = \frac{k_f A_m}{f_m} = \frac{peak\ frequency\ deviation}{message\ bandwidth} = \frac{\Delta f}{f_m}$
- In the figure below, we show a sinusoidal message signal $m(t)$ and the resulting FM signal $s(t)$.

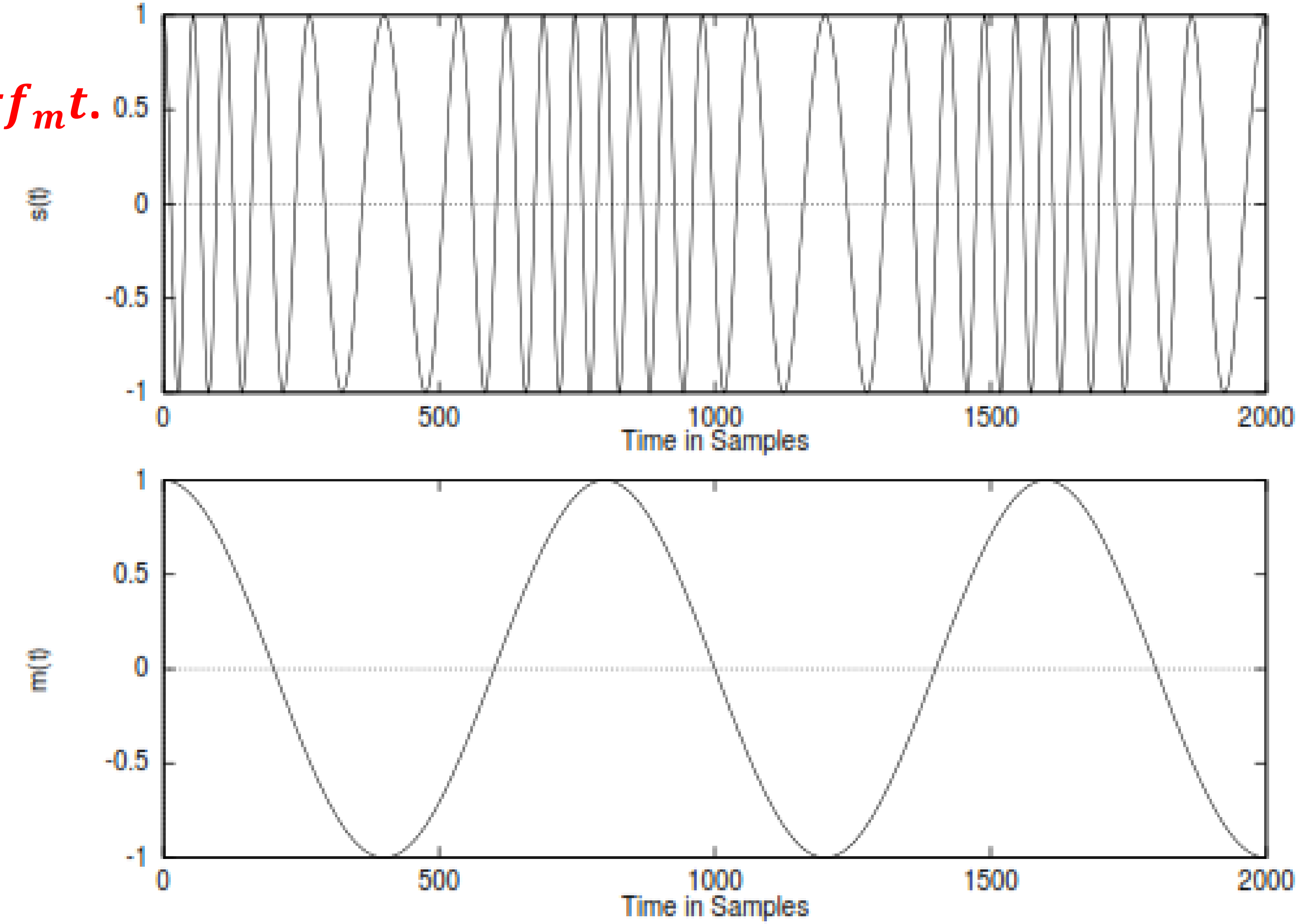


Single Tone Frequency Modulation

$$f_i = f_c + A_m k_f \cos 2\pi f_m t.$$

$$T = \frac{1}{f}$$

fc = 1 KHz,
fm = 100 Hz.



Spectrum of a Single-Tone FM Signal

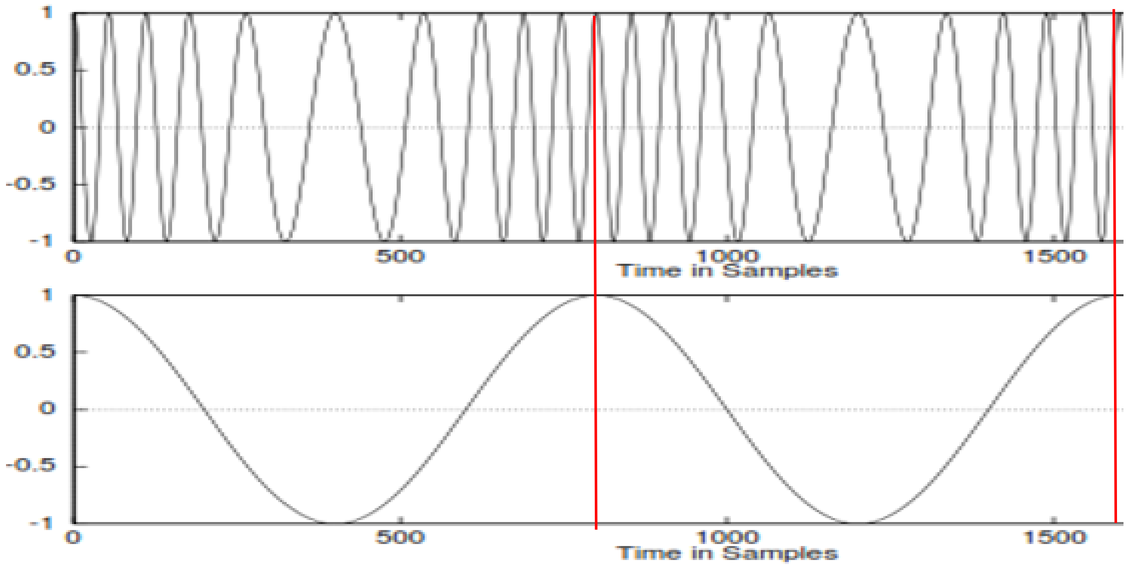
- The objective of this lecture is to find a meaningful definition of the bandwidth of an FM signal.
- To accomplish that, we need to find the spectrum of an FM signal with a single-tone test message signal.
- **Review of basic results from the previous lecture:**
- The expression for an angle modulated signal is: $\boldsymbol{s(t) = A_c \cos\left(2\pi f_c t + \theta(t)\right)}$
- The instantaneous frequency of s(t) is: $\boldsymbol{f_i(t) = f_c + \frac{1}{2\pi}\frac{d\theta(t)}{dt}}$
- For **phase modulation**:
  - $\theta(t) = k_p m(t)$, k_p in rad/volt.
  - $s(t)_{PM} = A_c \cos\left(2\pi f_c t + k_p m(t)\right)$.
  - $f_i(t) = f_c + \frac{k_p}{2\pi}\frac{dm(t)}{dt}$

Spectrum of a Single-Tone FM Signal

- **Review of basic results from the previous lecture:**
- The expression for an angle modulated signal is: $\boldsymbol{s(t) = A_c \cos\left(2\pi f_c t + \theta(t)\right)}$
- The instantaneous frequency of s(t) is: $\boldsymbol{f_i(t) = f_c + \frac{1}{2\pi}\frac{d\theta(t)}{dt}}$
- For **frequency modulation**:
  - $\theta(t) = 2\pi k_f \int_0^t m(\alpha)d\alpha;$ k_f in Hz/volt.
  - $s(t)_{FM} = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\alpha)d\alpha\right).$
  - $f_i(t) = f_c + k_f m(t);$
- When $\boldsymbol{m(t) = A_m \cos 2\pi f_m t}$
  - $f_i = f_c + A_m k_f \cos 2\pi f_m t;$
  - $s(t)_{FM} = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t A_m \cos \omega_m \alpha \, d\alpha\right) = \boldsymbol{A_c \cos\ (2\pi f_c t + \beta \sin 2\pi f_m t)}.$
- $\boldsymbol{\beta = \frac{k_f A_m}{f_m}} = \frac{peak\ frequency\ deviation}{message\ bandwidth} = \frac{\Delta f}{f_m};$ is the **FM modulation index,**

Spectrum of a Single-Tone FM Signal

- Let $m(t) = A_m \cos 2\pi f_m t$ be the test message signal, then the FM signal is
- $\boldsymbol{s(t) = A_c \cos(2\pi f_c t + \beta \sin 2\pi f_m t)}$
- $\boldsymbol{\beta = \frac{k_f A_m}{f_m}}$; **FM modulation index**
- $s(t)$ can be rewritten as:
- $s(t) = Re\{e^{j(2\pi f_c t + \beta \sin 2\pi f_m t)}\}$
- $\quad = Re\{e^{j(2\pi f_c t)} . e^{j(\beta \sin 2\pi f_m t)}\}$



- Remember that: $e^{j\theta} = cos\theta + jsin\,\theta$ and that $cos\theta = Re\{e^{j\theta}\}$
- The sinusoidal waveform ($\beta \sin 2\pi f_m t$) is periodic with period $T_m = \frac{1}{f_m}$. The exponential function $\boldsymbol{e^{j(\beta \sin 2\pi f_m t)}}$ is also periodic with the same period $T_m = \frac{1}{f_m}$
- $e^{j\beta \sin 2\pi f_m (t+T_m)} = e^{j\beta \sin 2\pi f_m t} . e^{j\beta \sin 2\pi f_m T_m} = e^{j\beta \sin 2\pi f_m t}$; $f_m T_m = 1 \Rightarrow$ $\sin(2\pi f_m T_m) = 0$;

Spectrum of a Single-Tone FM Signal

- $s(t) = A_c \, cos(\, 2\pi f_c t + \beta \, sin \, 2\pi f_m t \,)$
- $s(t) = Re\{e^{j(2\pi f_c t)} \; \boldsymbol{e^{j(\beta \sin 2\pi f_m t)}}\}$
- A periodic function g(t) can be expanded into a complex Fourier series as:
- $g(t) = \sum_{-\infty}^{\infty} C_n \, e^{jn\omega_m t} \;;$
- $C_n = \frac{1}{T_m} \int_0^{T_m} g(t) \, e^{-jn\omega_m t} \, dt$
- Now, let $\boldsymbol{g(t) = e^{j(\beta \sin 2\pi f_m t)}}$
- $C_n = \frac{1}{T_m} \int_0^{T_m} e^{j(\beta \sin 2\pi f_m t)} e^{-jn\omega_m t} \, dt$
- It turns out that the Fourier coefficients

$$\boldsymbol{C_n = J_n(\beta).}$$

- where $J_n(\beta)$ is the Bessel function of the first kind of order n (will describe it on next slide)
- Hence, $g(t) = \sum_{-\infty}^{\infty} J_n(\beta) \, e^{jn\omega_m t} \;;$
- Substituting $g(t)$ into $s(t)$, we get
- $s(t) = A_c \, Re\{e^{j(2\pi f_c t)} \; \sum_{-\infty}^{\infty} J_n(\beta) \, e^{jn\omega_m t} \, \}$

$$= A_c \;\; Re\{ \sum_{-\infty}^{\infty} J_n(\beta) e^{j2\pi(f_c + nf_m)t} \, \}$$

$$= A_c \sum_{-\infty}^{\infty} J_n(\beta) \cos(2\pi(f_c + nf_m)t)$$

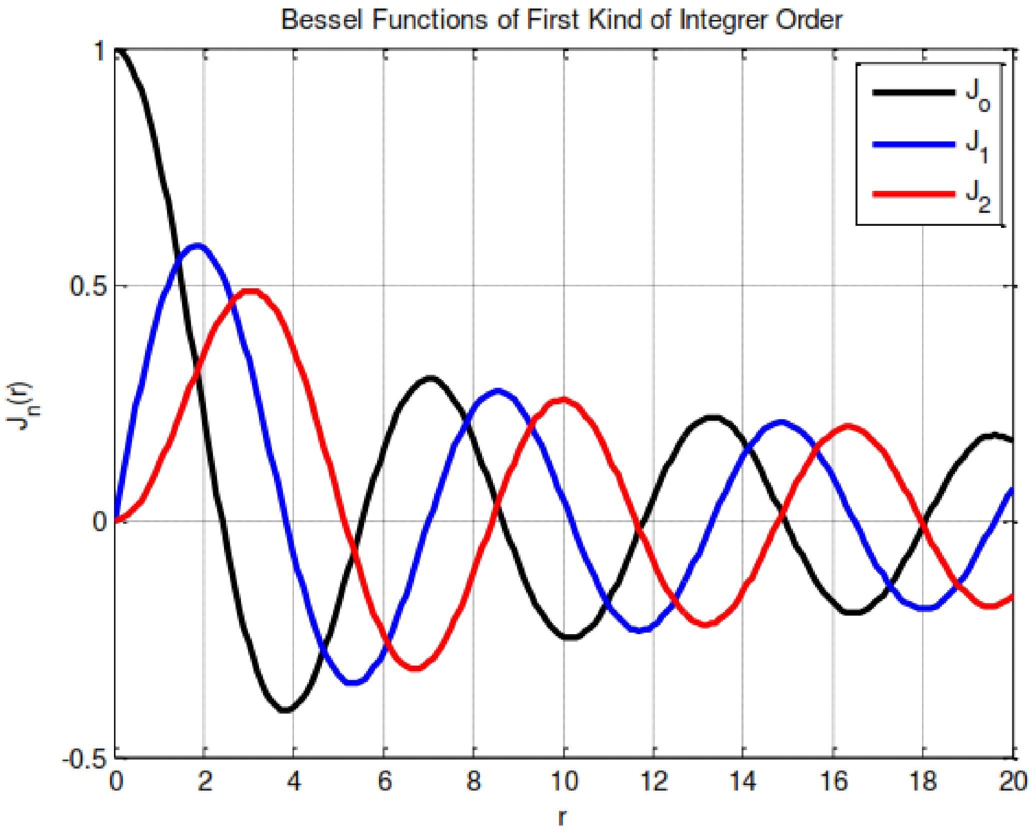
- Finally, the FM signal can be represented as
- $\boldsymbol{s(t) = A_c \, \sum_{-\infty}^{\infty} J_n(\beta) \cos(2\pi(f_c + nf_m)t)}$

Spectrum of a Single-Tone FM Signal

- **Bessel Functions**: The Bessel equation of order n is $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$
- This is a second order differential equation with variable coefficients. We can solve it using the power series method. The solution for each value of n is $J_n(x)$, the Bessel function of the first kind of order n. The figure, below, shows the first three Bessel functions.

Some Properties of Bessel Functions

- $J_n(x) = (-1)^n J_{-n}(x)$; relative to n
- $J_n(x) = (-1)^n J_n(-x)$; relative to x
- Recurrence formula $J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$.
- For small x, $J_n(x) \cong \frac{x^n}{2^n\ n!}$, $J_0(x) \cong 1, J_1(x) \cong \frac{x}{2}$.
- For large x: $J_n(x) \cong \sqrt{\frac{2}{\pi x}} \cos(x - \frac{\pi}{4} - \frac{n\pi}{2})$,
- $\sum_{n-\infty}^{\infty} (J_n(x))^2 = 1$, $for\ all\ x.$



The Fourier Series Representation of the FM Signal

- A single tone FM signal can be represented in a Fourier series as

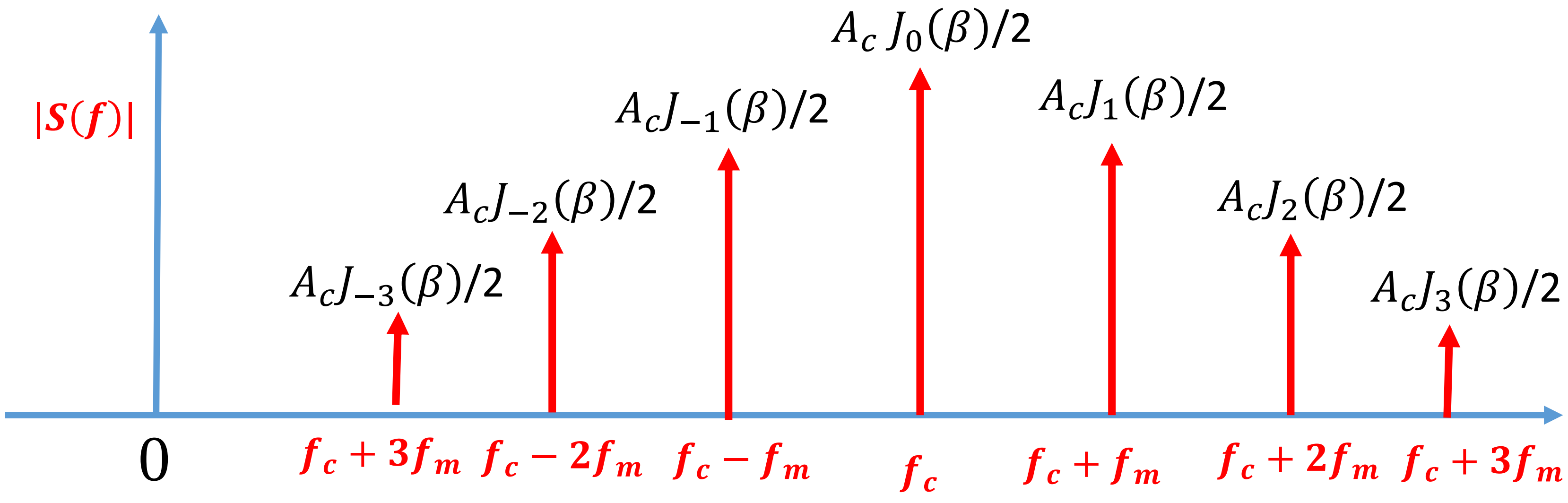
$$\boldsymbol{s(t) = A_c \sum_{-\infty}^{\infty} J_n(\beta) \cos(2\pi(f_c + nf_m)t)}$$

$$\boldsymbol{S(f) = A_c/2 \sum_{-\infty}^{\infty} J_n(\beta)[\delta(f - (f_c + nf_m) + \delta(f + (f_c + nf_m)]}$$

- The first few terms in this expansion are:
- $s(t) = A_c\, J_0(\beta)\cos(2\pi f_c t) + A_c J_1(\beta)\cos 2\pi(f_c + f_m)t + A_c J_{-1}(\beta)\cos 2\pi(f_c - f_m)t \ + A_c J_2(\beta)\cos 2\pi(f_c + 2f_m)t \ + A_c J_{-2}(\beta)\cos 2\pi(f_c - 2f_m)t + \cdots$
- $J_{-1}(\beta) = -J_1(\beta),\ J_{-2}(\beta) = J_2(\beta); J_{-3}(\beta) = -J_3(\beta); J_{-4}(\beta) = J_4(\beta);$

Remarks:

1. Spectral components are separated by fm.
2. The 98% power bandwidth is: $B_T = 2(\beta + 1)f_m$



The Fourier Series Representation of the FM Signal

- **Few remarks about the FM spectrum:**
- The FM signal $s(t) = A_c\, cos(\, 2\pi f_c t + \beta\, sin\, 2\pi f_m t\,)$ can be represented as
- $\boldsymbol{s(t) = A_c \sum_{-\infty}^{\infty} J_n(\beta) \cos(2\pi(f_c + nf_m)t)}$
- **s(t) =** $\boldsymbol{A_c J_0(\beta) \cos(2\pi(f_c)t)}$ **+**

  $\boldsymbol{A_c J_1(\beta) \cos(2\pi(f_c + f_m)t)}$ **+** $\boldsymbol{A_c J_{-1}(\beta) \cos(2\pi(f_c - f_m)t)}$

  + $\boldsymbol{A_c J_2(\beta) \cos(2\pi(f_c + 2f_m)t)}$ **+** $\boldsymbol{A_c J_{-1}(\beta) \cos(2\pi(f_c - 2f_m)t)}$

  + $\boldsymbol{A_c J_3(\beta) \cos(2\pi(f_c + 3f_m)t)}$ **+** $\boldsymbol{A_c J_{-3}(\beta) \cos(2\pi(f_c - 3f_m)t)}$ **+ …..**
- The FM signal consists of an infinite number of spectral components concentrated around f_c.
- Therefore, the theoretical bandwidth of the signal is infinity. That is to say, if we need to recover the FM signal without any distortion, all spectral components must be accommodated. This means that a channel with infinite bandwidth is needed. This is, of course, not practical since the frequency spectrum is shared by many users.
- In the following discussion we need to truncate the series so that, say 98%, of the total average power is contained within a certain bandwidth. But, first let us find the total average power using the series approach.

Power in the Spectral Components of s(t)

- A single tone FM signal $s(t) = A_c\, cos(\, 2\pi f_c t + \beta\, sin\, 2\pi f_m t\,)$ is expanded as:

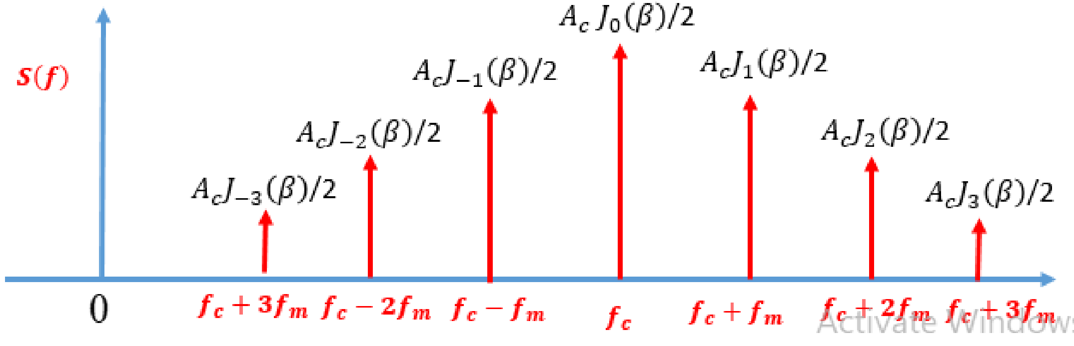
$$\boldsymbol{s(t) = A_c \sum_{-\infty}^{\infty} J_n(\beta)\cos(2\pi(f_c + nf_m)t)}; \quad \textbf{average power} = < \boldsymbol{s^2\ (t)} > = \boldsymbol{\frac{A_C{}^2}{2}}.$$

- Note that s(t) consists of an infinite number of Fourier terms, and the power in s(t) will be equal the power in the respective Fourier components
- Any term in s(t) takes the form: $A_c J_n(\beta)\cos(2\pi(f_c + nf_m)t)$
- The average power in this term is: $\frac{(A_c)^2(J_n(\beta))^2}{2}$
- Hence the total power in s(t) is
- $< s^2(t) > \ = \ \frac{A_C{}^2\, J_0{}^2(\beta)}{2} + \frac{A_C{}^2 J_1{}^2(\beta)}{2} + \frac{A_C{}^2 J_{-1}{}^2(\beta)}{2} + \frac{A_C{}^2 J_2{}^2(\beta)}{2} + \frac{A_C{}^2 J_{-2}{}^2(\beta)}{2} + \cdots$
- $= \frac{A_C{}^2}{2}\,\{J_0{}^2(\beta) + J_1{}^2(\beta) + J_{-1}{}^2\,(\beta) + J_2{}^2(\beta) + J_{-2}{}^2(\beta) + \cdots ..\}$
- $= \frac{A_C{}^2}{2}\,\{J_0{}^2(\beta) + 2J_1{}^2(\beta) + 2J_2{}^2(\beta) + \cdots ..\} = \frac{A_C{}^2}{2}$

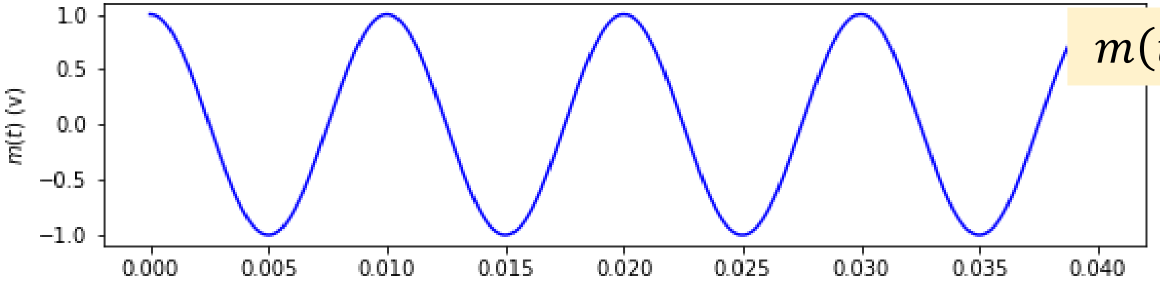
Remark: $\{\sum_{n=-\infty}^{\infty} J_n{}^2(\beta) = 1\}$, (A property of Bessel functions).

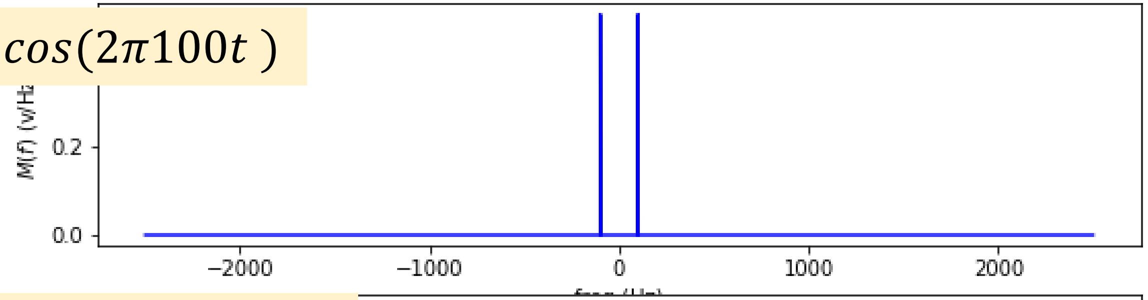
Example: 99% Power Bandwidth of an FM Signal

- Find the 99% power bandwidth of an FM signal when $\beta = 1$
- **Solution**: $s(t) = A_c \sum_{-\infty}^{\infty} J_n(\beta)\cos(2\pi(f_c + nf_m)t)$
- **Case a: $\beta = 1$ (wideband FM)**
- The first five terms corresponding to $\beta = 1$ (obtained from the table) are
- $J_0(1) = 0.7652$, $J_1(1) = 0.4401$, $J_2(1) = 0.1149$, $J_3(1) = 0.01956$, $J_4(1) = 0.002477$
- The power in s(t) is $< S^2(t) > = \frac{A_C{}^2}{2}$
- Let us try to find the average power in the terms at (f_c), (f_c+f_m), (f_c-f_m), (f_c+2f_m), (f_c-2f_m)
- f_c : $\frac{A_C{}^2 J_0{}^2(\beta)}{2}$; $f_c + f_m$: $\frac{A_C{}^2 J_1{}^2(\beta)}{2}$; $f_c - f_m$: $\frac{A_C{}^2 J_{-1}{}^2(\beta)}{2}$; $f_c + 2f_m$: $\frac{A_C{}^2 J_2{}^2(\beta)}{2}$; $f_c - 2f_m$: $\frac{A_C{}^2 J_{-2}{}^2(\beta)}{2}$
- The average power in the five spectral components is the sum
- $P_{av} = \frac{A^2{}_c}{2}[J_0{}^2(1) + 2J_1{}^2(1) + 2J_2{}^2(1)]$; $P_{av} = \frac{A^2{}_c}{2}[(0.7652)^2 + 2 * (0.4401)^2 + (0.1149)^2] = 0.9993\,\frac{A^2{}_c}{2}$
- Hence, these terms contain 99.9 % of the total power.
- Therefore, the 99.9 % power bandwidth is
- $BW = (f_c + 2f_m) - (f_c - 2f_m) = 4f_m$

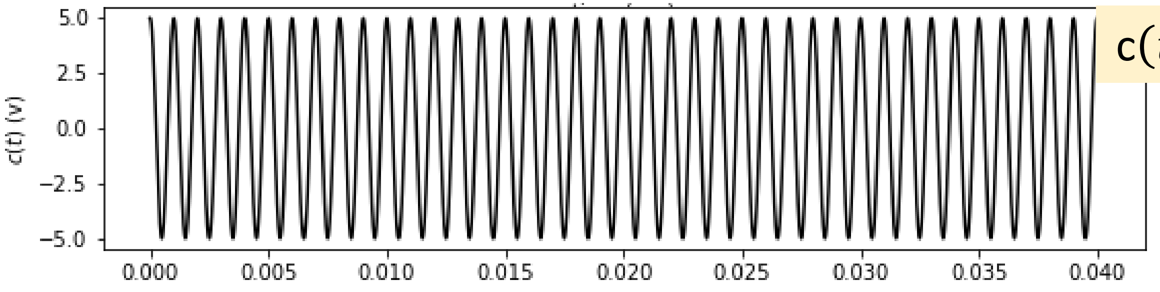


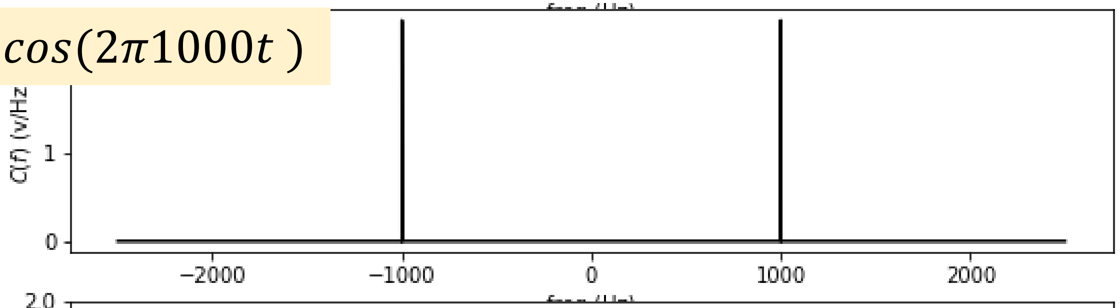
FM in the time and frequency domains: $\beta = 1$



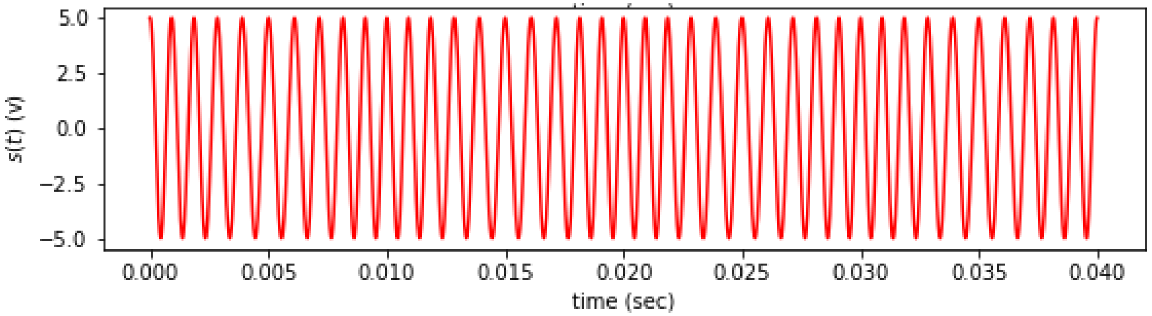


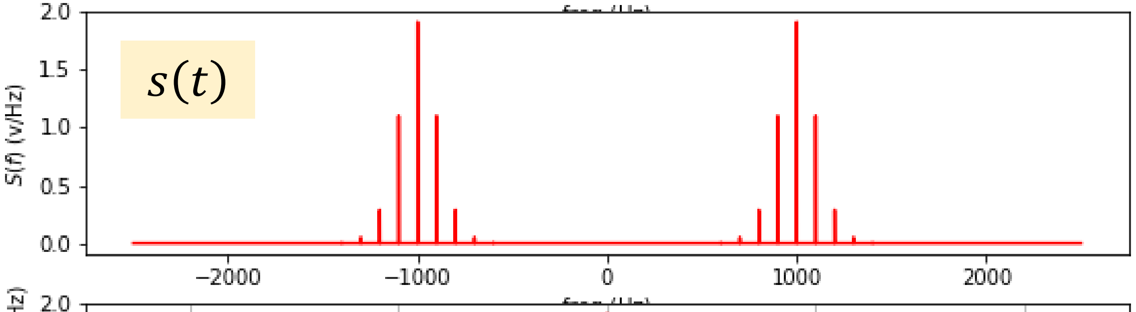
$m(t) = cos(2\pi 100t)$



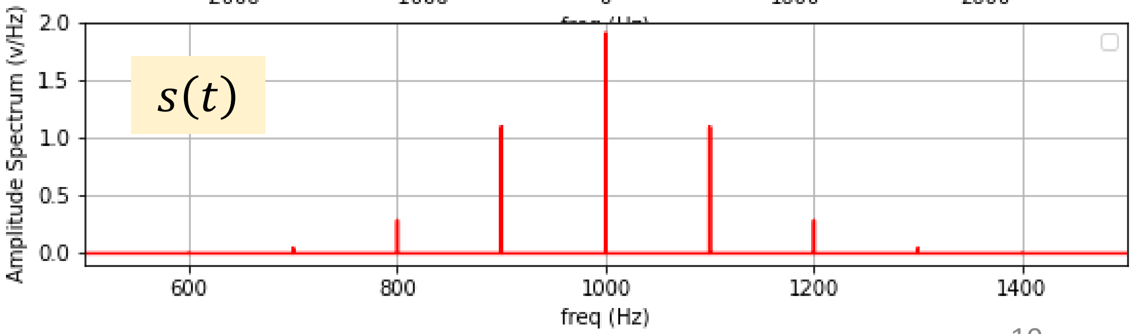


$c(t) = 5cos(2\pi 1000t)$





$s(t)$



$s(t)$

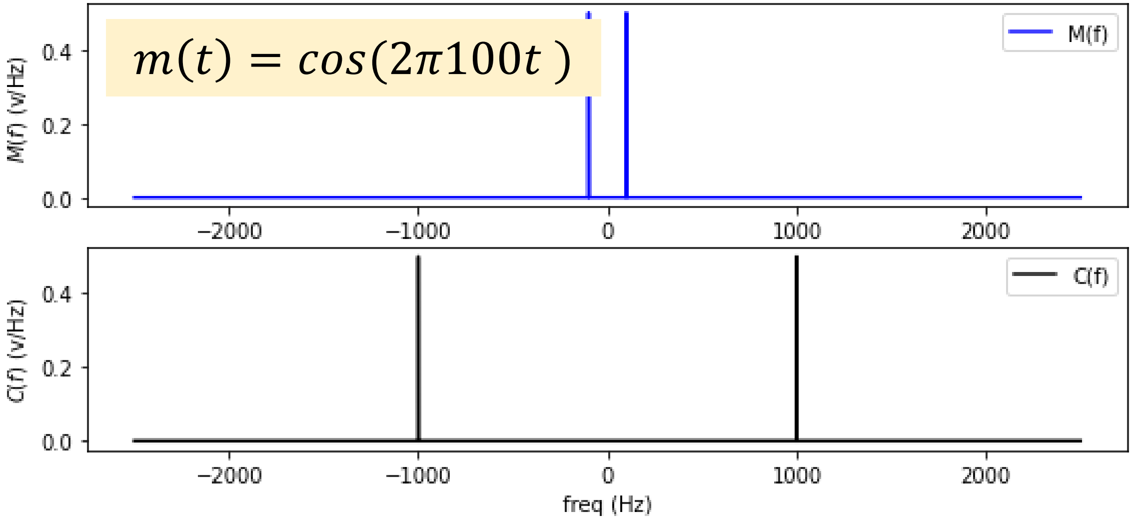
Kf = 100 Hz/V $s(t) = 5\, cos(\, 2\pi f_c t + 5\, sin\, 2\pi 100t\,)$

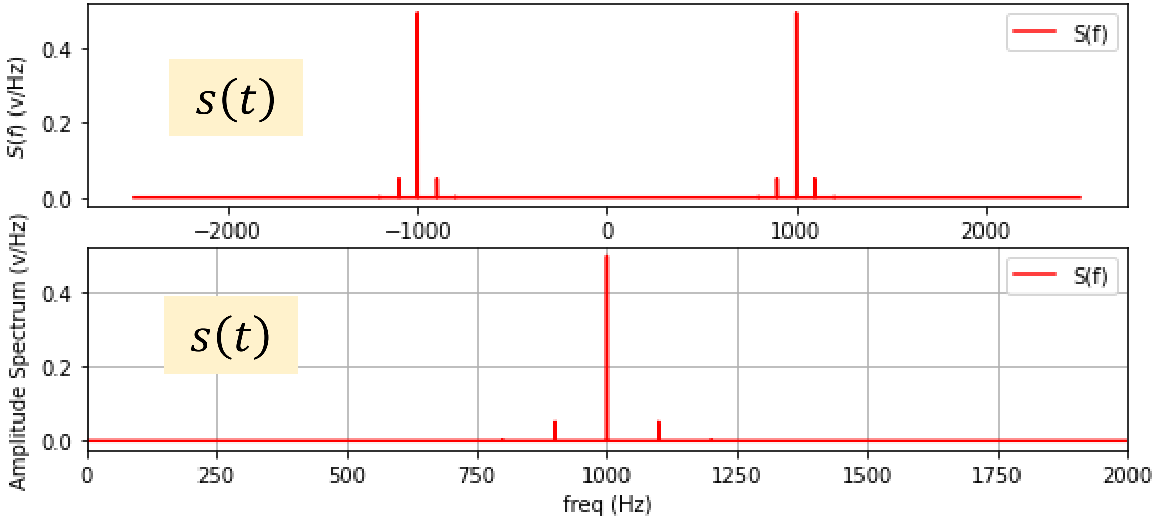
$$\boldsymbol{BW = (f_c + 2f_m) - (f_c - 2f_m) = 4f_m = 400\ Hz}$$

Example: 99% Power Bandwidth of an FM Signal

- Find the 99% power bandwidth of an FM signal when $\beta = 0.2$
- **Solution**: $s(t) = A_c \sum_{-\infty}^{\infty} J_n(\beta) \cos(2\pi(f_c + nf_m)t)$
- **Case b: $\boldsymbol{\beta = 0.2}$ (Narrowband FM)**
- For $\beta = 0.2,\ J_0(0.2) = 0.99,\ J_1(0.2) = 0.0995,\ \ J_2(0.2) = 0.00498335$
- The power in the carrier and the two sidebands at $(f_c\ , f_c + f_m, f_c - f_m)$ is
- $P = \frac{A^2{}_c}{2}\left[J_0{}^2(0.2) + 2J_1{}^2(0.2)\right]$
- $P = \frac{A^2{}_c}{2}[0.9999]$
- Therefore, 99.99% of the total power is found in the carrier and the two sidebands.
- The 99% bandwidth is: $\boldsymbol{B.W = (f_c + f_m) - (f_c - f_m) = 2f_m}$

Narrow-Band FM: $\beta = 0.2$





$\text{c}(t) = cos(2\pi 1000t\ )$

$s(t) = cos(\ 2\pi f_c t + 0.2\, sin\, 2\pi 100t\ )$

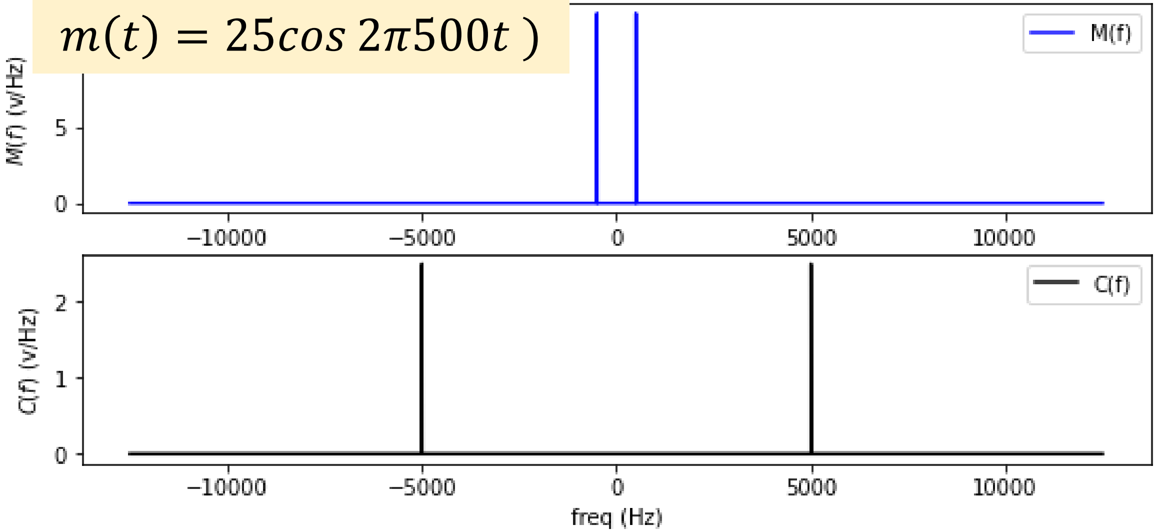
Spectrum is similar to that of normal AM

$\boldsymbol{BW = (f_c + f_m) - (f_c - f_m) = 2f_m = 200\ Hz}$

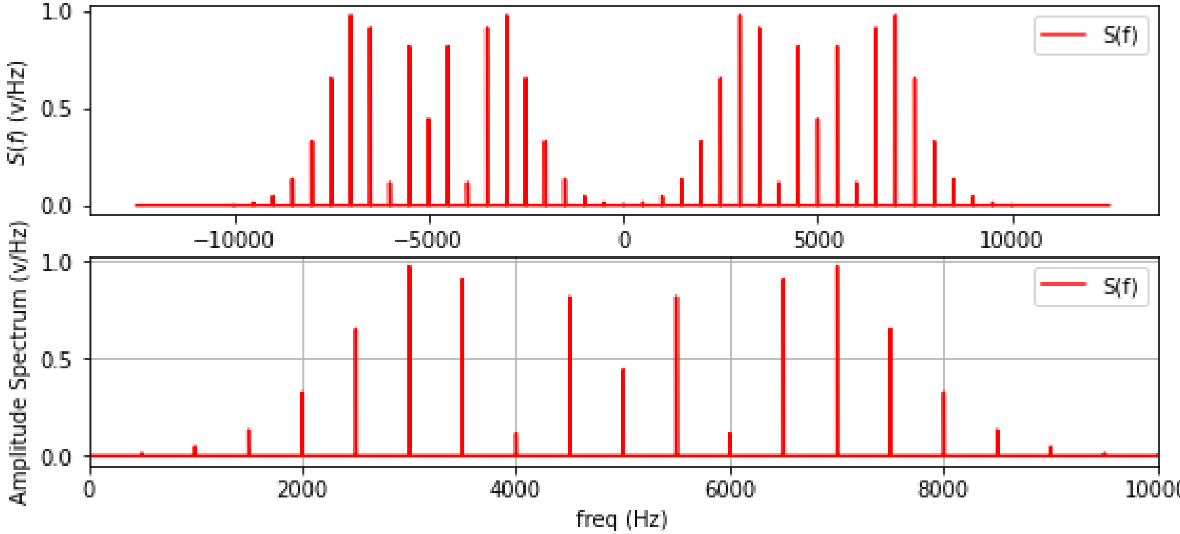
Kf = 20 Hz/V

Wideband-Band FM : $\beta = 5$

$m(t) = 25cos\,2\pi 500t\,)$



$c(t) = 5cos\,2\pi 5000t\,)$



$s(t) = 5cos(\,2\pi f_c t + 5\,sin\,2\pi 500t\,)$

$$BW = (f_c + 6f_m) - (f_c - 6f_m) = 12f_m = 6000\,Hz$$

Kf = 100 Hz/V

Carson's Rule

- A 98% power B.W of an FM signal can be estimated using Carson's rule
- $\boldsymbol{B_T = 2(\beta + 1)f_m}$
- The rule works well when the message signal is continuous (Cannot be used when the message contains discontinuities as in the case of a square function).
- **Example**: Find the bandwidth of the FM signal
- $s(t) = A_c \, cos(\, 2\pi f_c t + sin \, 2\pi f_m t \,)$
- **Solution**: $B_T = 2(\beta + 1)f_m = 2(1 + 1)f_m = 4f_m$
- **Example**: Find the bandwidth of the FM signal

  $s(t) = A_c \, cos(\, 2\pi f_c t + 5 \, sin \, 2\pi f_m t \,)$
- **Solution**: $B_T = 2(\beta + 1)f_m = 2(5 + 1)f_m = 12f_m$
- **Remark**: Same result as was obtained using the spectral analysis.

Table of Bessel Functions

β	J0(β)	J1(β)	J2(β)	J3(β)	J4(β)	J5(β)
0	1	0	0	0	0	0
0.1	0.9975	0.0499	0.0012	0.0000	0.0000	0.0000
0.2	0.9900	0.0995	0.0050	0.0002	0.0000	0.0000
0.3	0.9776	0.1483	0.0112	0.0006	0.0000	0.0000
0.4	0.9604	0.1960	0.0197	0.0013	0.0001	0.0000
0.5	0.9385	0.2423	0.0306	0.0026	0.0002	0.0000
0.6	0.9120	0.2867	0.0437	0.0044	0.0003	0.0000
0.7	0.8812	0.3290	0.0588	0.0069	0.0006	0.0000
0.8	0.8463	0.3688	0.0758	0.0102	0.0010	0.0001
0.9	0.8075	0.4059	0.0946	0.0144	0.0016	0.0001
1	0.7652	0.4401	0.1149	0.0196	0.0025	0.0002

Generation of an FM Signal
Lecture Outline

- In this lecture, we present two methods for the generation of a frequency modulated signal:
  - The direct method, which uses a voltage controlled oscillator
  - The indirect method, in which a narrow band FM is generated first, then frequency multipliers are used to produce the desired wideband FM.
- Both methods are analyzed in detail.
- The operation of the varactor diode is briefly described.

Review: Basics of Angle Modulation

- The expression for an angle modulated signal is: $\boldsymbol{s(t) = A_c \cos(2\pi f_c t + \theta(t))}$
- The instantaneous frequency of s(t) is:

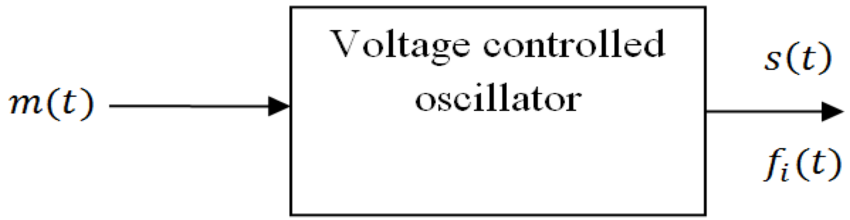
$$\boldsymbol{f_i(t) = f_c + \frac{1}{2\pi}\frac{d\theta(t)}{dt}}$$

- For **phase modulation**:
  - $\theta(t) = k_p m(t)$, k_p in rad/volt.
  - $s(t)_{PM} = A_c \cos\left(2\pi f_c t + k_p m(t)\right)$
  - $f_i(t) = f_c + \frac{k_p}{2\pi}\frac{dm(t)}{dt}$
- For **frequency modulation**:
  - $f_i(t) = f_c + k_f m(t);$
  - $\theta(t) = 2\pi k_f \int_0^t m(\alpha)d\alpha;$ k_f in Hz/volt.
  - $s(t)_{FM} = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\alpha)d\alpha\right).$

- When $\boldsymbol{m(t) = A_m \cos 2\pi f_m t}$
  - $f_i = f_c + A_m k_f \cos 2\pi f_m t;$
  - $s(t)_{FM} = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t A_m \cos \omega_m \alpha \, d\alpha\right)$
  - = $\boldsymbol{A_c \cos\ (2\pi f_c t + \beta \sin 2\pi f_m t)}$.
- $\boldsymbol{\beta = \frac{k_f A_m}{f_m}} = \frac{peak\ frequency\ deviation}{message\ bandwidth} = \frac{\Delta f}{f_m};$
- $\boldsymbol{\beta}$: is the **FM modulation index,**
- When $m(t) = A_m \cos 2\pi f_m t$, FM signal can be represented as
- $\boldsymbol{s(t) = A_c \sum_{-\infty}^{\infty} J_n(\beta) \cos(2\pi(f_c + n f_m)t)}$
- Carson's rule: $\boldsymbol{B_T = 2(\beta + 1) f_m}$
- When $\boldsymbol{\beta \ll 1}$, the FM is termed narrow band (the BW is comparable to the BW of AM)
- Otherwise, it is termed a wideband FM. Here the BW. Is much larger than that of the AM signal.

Generation of an FM Signal

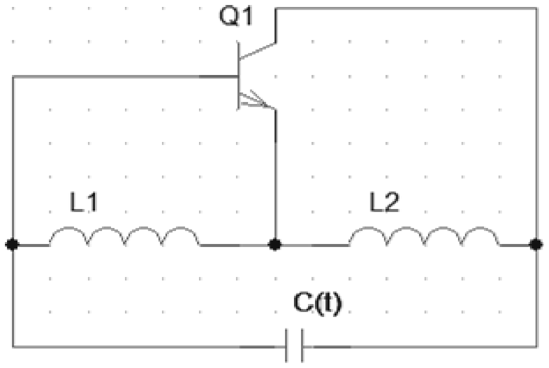
- **Direct Method for Generating an FM Signal**
- In a direct FM system, the instantaneous frequency of the carrier is varied in accordance with a message signal by means of a voltage-controlled oscillator (VCO). The voltage – frequency characteristic of a VCO is given by
- $\boldsymbol{f_i = f_c + k_f\, m(t)}$
- k_f : proportionality constant Hz/V
- A schematic diagram of a VCO is shown in the figure
- A realization of the CVO may be obtained by considering an oscillator (like the Hartley oscillator) shown on the next slide in which a varactor (voltage variable capacitor) is used. **A varactor diode is a semiconductor diode whose junction capacitance varies linearly with the applied voltage when the diode is reverse biased**



$$\boldsymbol{f_i = f_c + k_f\, m(t)}$$

Direct Method for Generating an FM Signal

- For the Hartley oscillator shown, the frequency of oscillation is $\boldsymbol{f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)C(t)}}}$
- Let $\boldsymbol{C(t) = C_0 - k\,m(t)}$ (A varactor diode operating in the reverse bias region can act like a variable capacitor); k is a constant,
- When m(t) = 0, $C(t) = C_0$, and $f_c = \frac{1}{2\pi\sqrt{(L_1+L_2)C_0}}$
- When m(t) has a finite value, the frequency of oscillation is
- $f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)(C_0-k\,m(t))}} = \frac{1}{2\pi\sqrt{C_0(L_1+L_2)}\sqrt{(1-k\,m(t)/C_0)}}$
- $= f_c\left(1 - \frac{k\,m(t)}{C_0}\right)^{-1/2}$
- When $\frac{k\,m(t)}{C_0} \ll 1$, we can make the approximation (using $[(1+x)^n \cong 1+nx]$ when x is small)
- $f_i(t) = f_c\left(1 + \frac{k\,m(t)}{2C_0}\right) = \boldsymbol{f_c + k_f\,m(t)}$
- Here it is clear that the instantaneous frequency varies linearly with the message signal.
- **Remark**: Direct method of FM generation is very simple and cheap process, but this method can't be used for broadcast application because the LC oscillator used in this method is not very stable. Its frequency depends upon various parameters such as temperature, device aging etc.



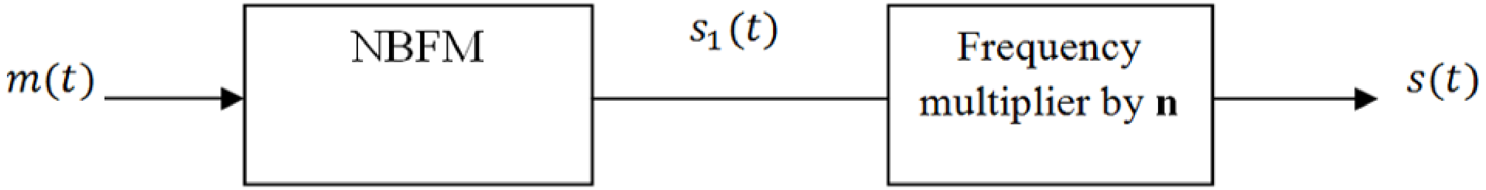
$\boldsymbol{C(t) = C_0 - k\,m(t)}$

Indirect Method for Generating an FM Signal

- A wideband FM can be generated indirectly using the block diagram below. First, a narrowband FM is generated. Then, the wideband FM is obtained by using frequency multiplication. Next, we analyze the operation of this modulator.
- Let $m(t) = A_m \cos 2\pi f_m t$ be the baseband signal, then
- $s_1(t) = A_c \cos(2\pi f_c' t + \beta' \sin 2\pi f_m t)$; $\beta' = \frac{k_f A_m}{f_m}$ is a NBFM with $\beta' \ll 1$.
- The frequency of $s_1(t)$ is $f_i' = f_c' + k_f A_m \cos 2\pi f_m t$
- Multiplying f_i by n (through frequency multiplication), we get the frequency of $s(t)$ as

$\boldsymbol{f_i' = f_c' + k_f A_m cos 2\pi f_m t}$ $\boldsymbol{f_i = n f_c' + n k_f A_m cos 2\pi f_m t}$

- $f_i = n f_c' + n k_f A_m cos 2\pi f_m t$



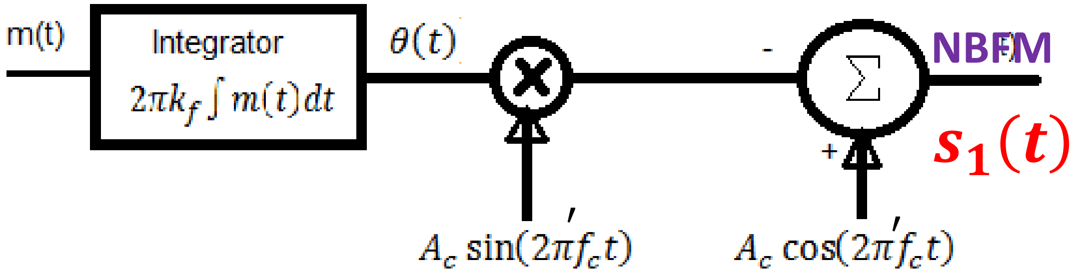
- The result is
- $s(t) = A_c \cos[2\pi(n f_c')t + n\beta' \sin 2\pi f_m t] = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$
- Where $\beta = n\beta'$ is the desired modulation index of WBFM
- $f_c = n f_c'$ is the desired carrier frequency of WBFM

$\boldsymbol{f_c' = 1KHz, \beta' = 0.2}$
$\boldsymbol{f_c = 10KHz, \beta = 2 \Rightarrow n = 10}$

Generation of an FM Signal: The NBFM

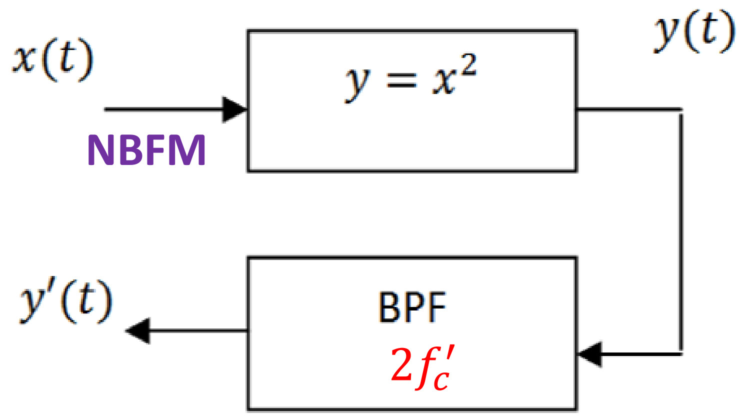
- Consider an FM signal
- $s_1(t) = A_c \cos(2\pi f_c' t + 2\pi k_f \int m(t)dt)$
- Assuming $\mathrm{m(t)} = A_m cos2\pi f_m t$,
- $\boldsymbol{s_1(t) = A_c\, cos(2\pi f'_c t + \beta' sin(2\pi f_m t))}$
- $s_1(t)$ can be expanded as
- $s_1(t) = A_c \cos(2\pi f_c' t) \cos\big(\theta(t)\big) - A_c \sin(2\pi f_c' t) \sin\big(\theta(t)\big)$
- When $|\theta(t)| = |\beta' \sin(2\pi f_m t)| \ll 1$, $cos\ \theta \cong 1$, $\sin(\theta) \cong \theta$.
- $s_1(t)$, termed narrowband, can be approximated as
- $s_1(t) \cong A_c \cos(2\pi f_c' t) - A_c \theta \sin(2\pi f_c' t)$
- $s_1(t) = A_c \cos(2\pi f_c' t) - A_c \beta' \sin(2\pi f_m t) \sin(2\pi f_c' t)$

$\boldsymbol{s_1(t) = A_c\, cos(2\pi f'_c t + \beta' sin(2\pi f_m t))}$



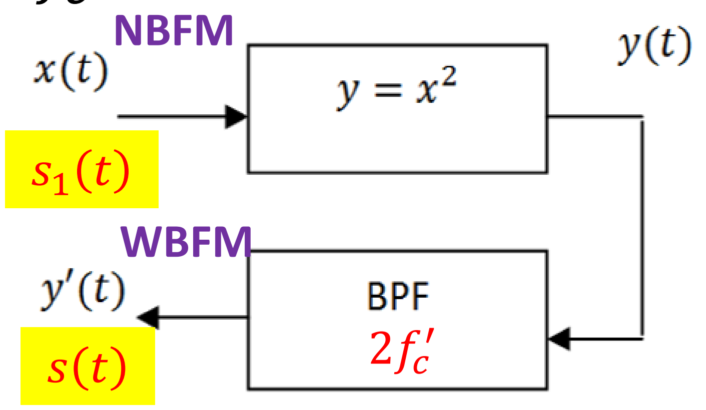
Generation of an FM Signal: Frequency Multiplication

- **Frequency Multiplier**: It is a device for which the frequency of the output signal is an integer multiple of the frequency of the input signal. It is primarily a nonlinear characteristic followed by a band pass filter. Now we illustrate the operation of this device.
- **The Square Law Device:** Let the input be an FM signal of the form:
- $x(t) = A_c \cos(2\pi f_c' t + \beta' \sin 2\pi f_m t) = A_c \cos(\phi)$
- The output of the square law characteristic is:
- $y(t) = x(t)^2 = A_c^2 cos^2(\phi) = \frac{A_c^2}{2}[1 + \cos(2\phi)]$
- $= \frac{A_c^2}{2} + \frac{A_c^2}{2}\cos(2\phi)$
- $= \frac{A_c^2}{2} + \frac{A_c^2}{2} cos[2\pi(2f_c')t + 2\beta' \sin(2\pi f_m t)]$



Generation of an FM Signal: Frequency Multiplication

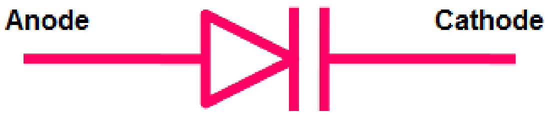
- $\mathrm{y(t)} = \frac{A_c^2}{2} + \frac{A_c^2}{2} cos[2\pi(2f_c')t + 2\beta' \sin(2\pi f_m t)];$
- If $y(t)$ is passed through a BPF of center frequency $2f_c'$, then the DC term will be suppressed and the filter output is



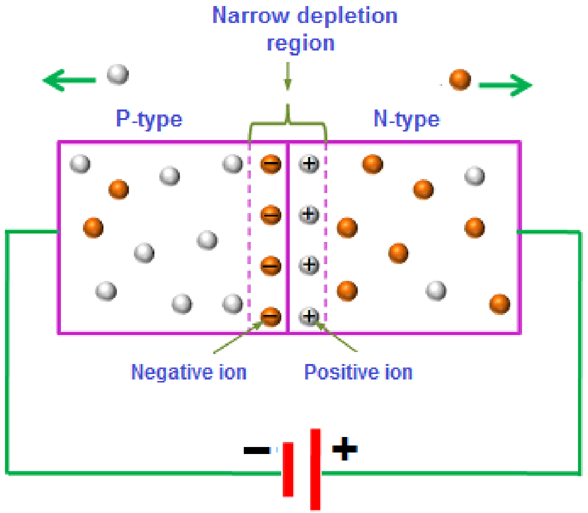
- $y'(t) = \frac{A_c^2}{2} cos[2\pi(2f_c')t + 2\beta' \sin(2\pi f_m t)]$
- $\boldsymbol{y'(t) = \frac{A_c^2}{2} cos[2\pi(f_c)t + \beta sin(2\pi f_m t)]}$
- As can be seen from this result, the output is a signal with twice the frequency of the input signal and a modulation index twice that of the input.
- $\boldsymbol{f_c = 2f_c'; \beta = 2\beta'}$
- To get frequency multiplication higher than two, a cascade of units, similar to what was described above, can be formed with the number of stages that achieve the desired carrier frequency and modulation index.

The Varactor Diode

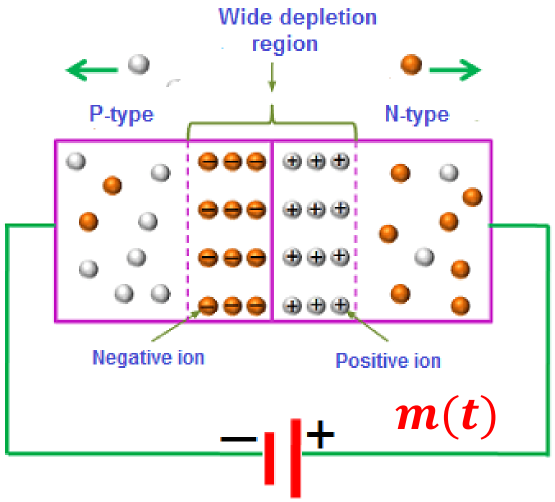
- **Definition:** The diode whose internal capacitance varies with the variation of the reverse voltage is known as the Varactor diode. The varactor diode always works in reverse bias, and it is a voltage-dependent semiconductor device.
- The Varactor diode is made up of n-type and p-type semiconductor material. In an n-type semiconductor material, the electrons are the majority charge carrier and in the p-type material, the holes are the majority carriers. When the p-type and n-type semiconductor material are joined together, the p-n junction is formed, and the depletion region is created at the PN-junction. The positive and negative ions make the depletion region.
- **Reference**: https://circuitglobe.com/varactor-diode.html



Symbol



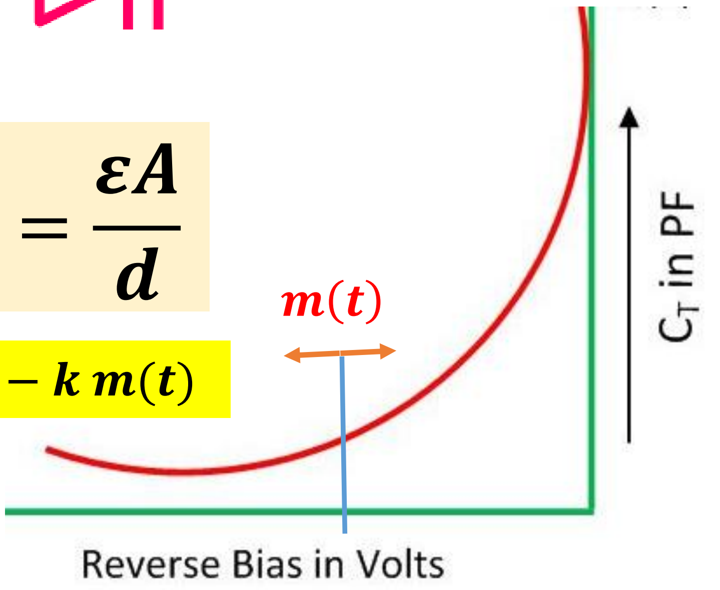
Large C: Low reverse bias voltage



Small C: High reverse bias voltage

$$C = \frac{\varepsilon A}{d}$$

$$C(t) = C_0 - k\,m(t)$$



Demodulation of an FM Signal
Lecture Outline

- In this lecture, we present two methods for the demodulation of a frequency modulated signal:
  - The discriminator, which is a differentiator followed by an envelope detector.
  - The phase locked loop.
- Both methods are analyzed in detail.
- The time response of the phase locked loop is analyzed in the transient and steady state conditions.
- The frequency response of a first order PLL is derived.
- The concept of pre-emphasis and de-emphasis in FM is introduced.

Review: Basics of Angle Modulation

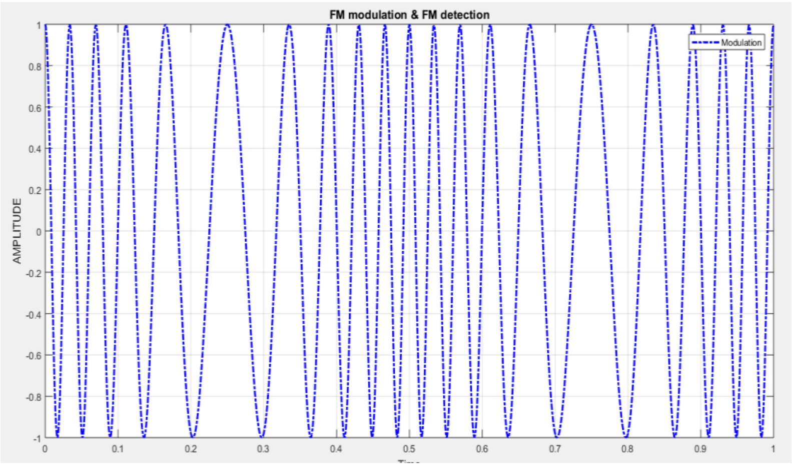
- The expression for an angle modulated signal is: $\boldsymbol{s(t) = A_c \cos\left(2\pi f_c t + \theta(t)\right)}$
- The instantaneous frequency of s(t) is: $\boldsymbol{f_i(t) = f_c + \frac{1}{2\pi}\frac{d\theta(t)}{dt}}$
- For **frequency modulation**:
  - $f_i(t) = f_c + k_f m(t);$ $\quad \Rightarrow \boldsymbol{m(t) = (f_i(t) - f_c)/k_f}$; Key Demodulation Concept
  - $\frac{1}{2\pi}\frac{d\theta(t)}{dt} = k_f m(t);\ \Rightarrow\ \theta(t) = 2\pi k_f \int_0^t m(\alpha)d\alpha;$ k_f in Hz/volt.
  - $s(t)_{FM} = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\alpha)d\alpha\right).$
- For **phase modulation**:
  - $\theta(t) = k_p m(t),$ k_p in rad/volt.
  - $s(t)_{PM} = A_c \cos\left(2\pi f_c t + k_p m(t)\right)$
  - $f_i(t) = f_c + \frac{k_p}{2\pi}\frac{dm(t)}{dt};$ $\quad \Rightarrow \boldsymbol{m(t) = \int_0^t 2\pi(f_i(\alpha) - f_c)d\alpha}$

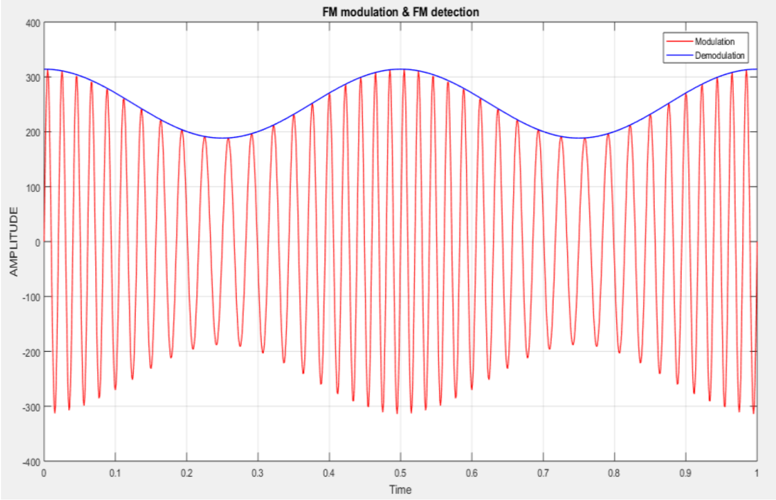
Demodulation of an FM Signal: The Discriminator

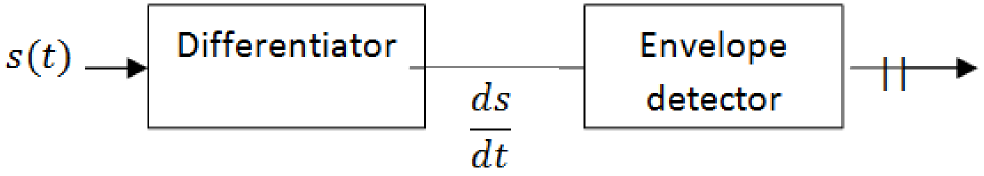
- An FM signal may be demodulated by means of what is called a ***discriminator***.
- One realization of a discriminator is a differentiator followed by an envelope detector, as illustrated in the figure. The operation of this discriminator can be explained as follows
- Let $s(t) = A_c \cos\big(\omega_c t + \theta(t)\big)$; $\theta(t) = 2\pi k_f \int_0^t m(\alpha)d\alpha,\ \theta(t) = k_p m(t)$
- $\dfrac{ds(t)}{dt} = -A_c\left(\omega_c + \dfrac{d\theta}{dt}\right)\sin\big(\omega_c t + \theta(t)\big)$
- The output of the envelope detector is $A_c\left|\left(\omega_c + \dfrac{d\theta}{dt}\right)\right| = A_c\omega_c + A_c\dfrac{d\theta}{dt}$
- The capacitor blocks the DC term and so output is:

$$\boldsymbol{V_0 = A_c \frac{d\theta}{dt}}$$

- If s(t) is an FM signal, then $\boldsymbol{V_0 = 2\pi k_f\, A_c m(t)}$
- If s(t) is a PM signal, then $\boldsymbol{V_0 = k_p \frac{dm(t)}{dt} \Rightarrow m(t) = k_p \int V_0\,(t)dt}$
- A typical FM signal and its derivative are shown in the figure.
- Next, we review the envelope detector and explain how a differentiator is implemented.







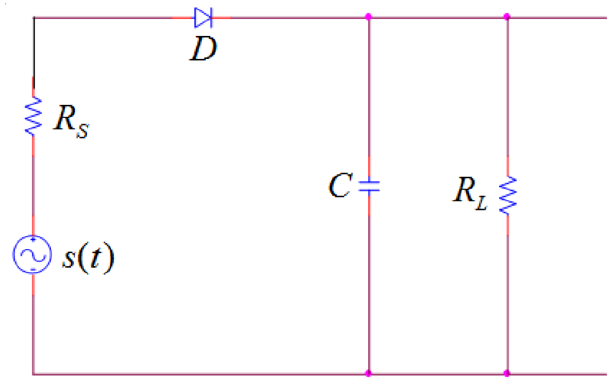
A Simple Practical Envelope Detector

The Ideal Envelope Detector: The ideal envelope detector responds to the envelope of the signal, but is insensitive to phase variation. If

$$s(t) = A_C\left(1 + k_a m(t)\right)\cos 2\pi f_C t$$

then, the output of the ideal envelope detector is $y(t) = A_C\left|1 + k_a m(t)\right|$

- A practical envelope detector consists of a diode followed by an RC circuit that forms a low pass filter.
- The operation of the envelope detector was described in a previous lecture.

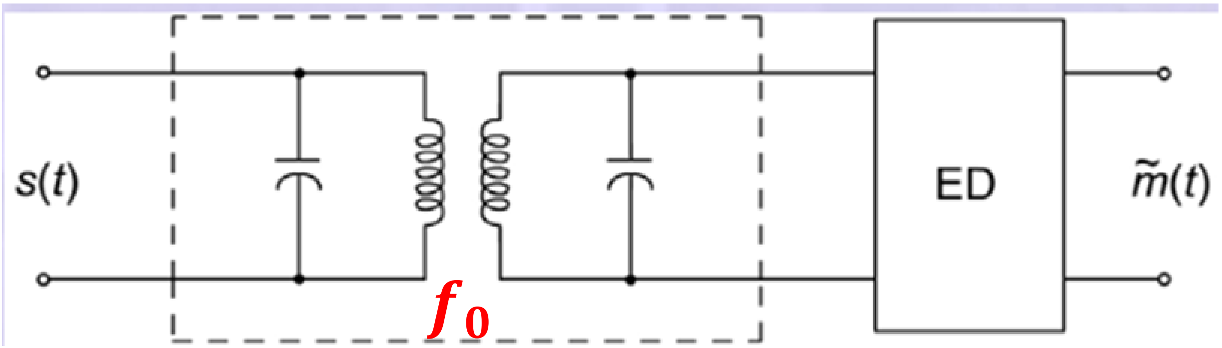


Demodulation of an FM Signal

- **Realization of the Differentiator**: From the properties of Fourier transform, we know that if

$$\Im\{g(t)\} = G(f), \quad \text{then} \quad \Im\left\{\frac{dg(t)}{dt}\right\} = j2\pi f G(f)$$

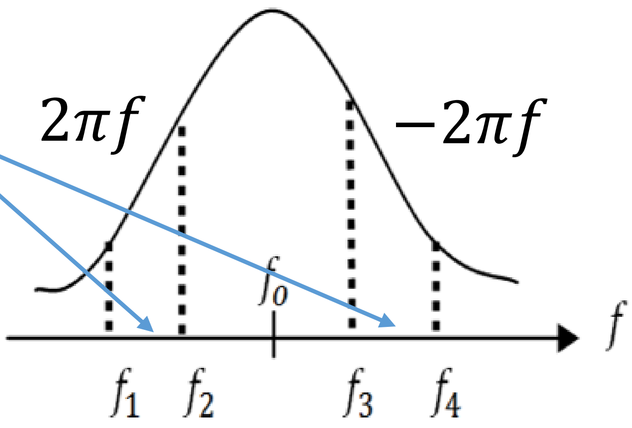
- This means that multiplication by $j2\pi f$ in the frequency domain amounts to differentiating the signal in the time-domain. Hence, we need a circuit whose frequency response is linear in f to perform time differentiation. A circuit that performs this task is a tuned circuit, provided that the signal frequency variation falls within the linear part of the characteristic, i.e., either between (f_1, f_2) or (f_3, f_4).
- The circuit below is a realization of an FM demodulator. The primary and secondary tuned circuits perform the task of differentiation, while the envelope detector extracts the envelope, which is supposed to be proportional to the message signal m(t)



Primary circuit tuned to f_c Secondary circuit tuned to $f_0 > f_c$

$$f_i(t) = f_c + k_f m(t).$$

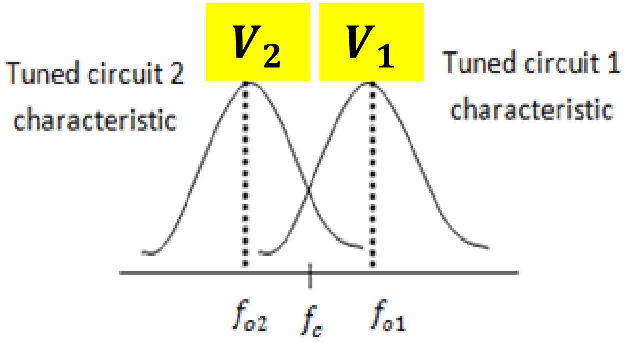
FM signal carrier should fall within these bands

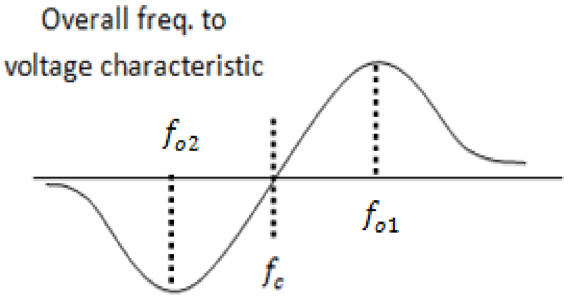


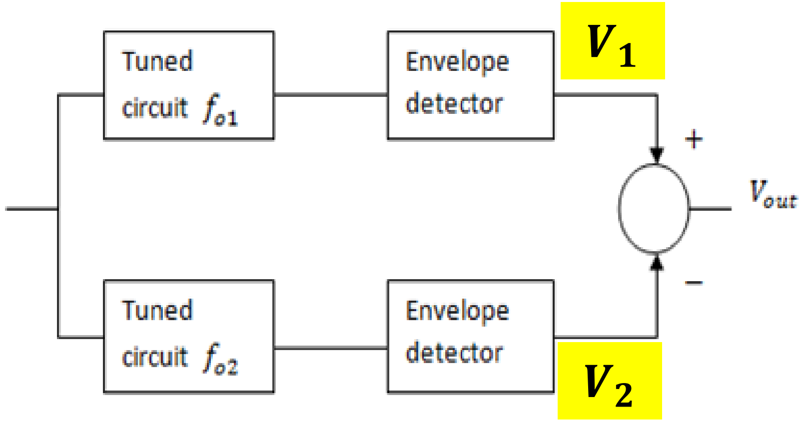
Demodulation of an FM Signal

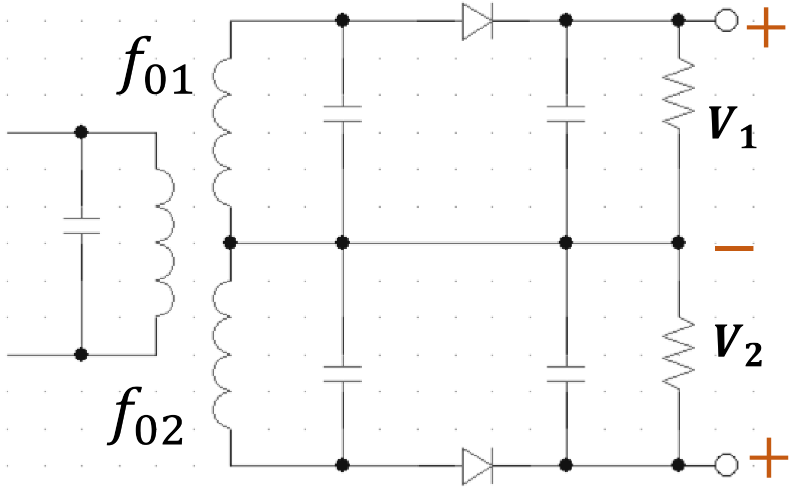
Balanced Slope Detector

- To extend the dynamic range of the differentiating circuit, two tuned circuits with center frequencies f_{01} and f_{02} are used as shown in the figure
- This circuit has a wider range of linear frequency response
- No DC blocking is necessary









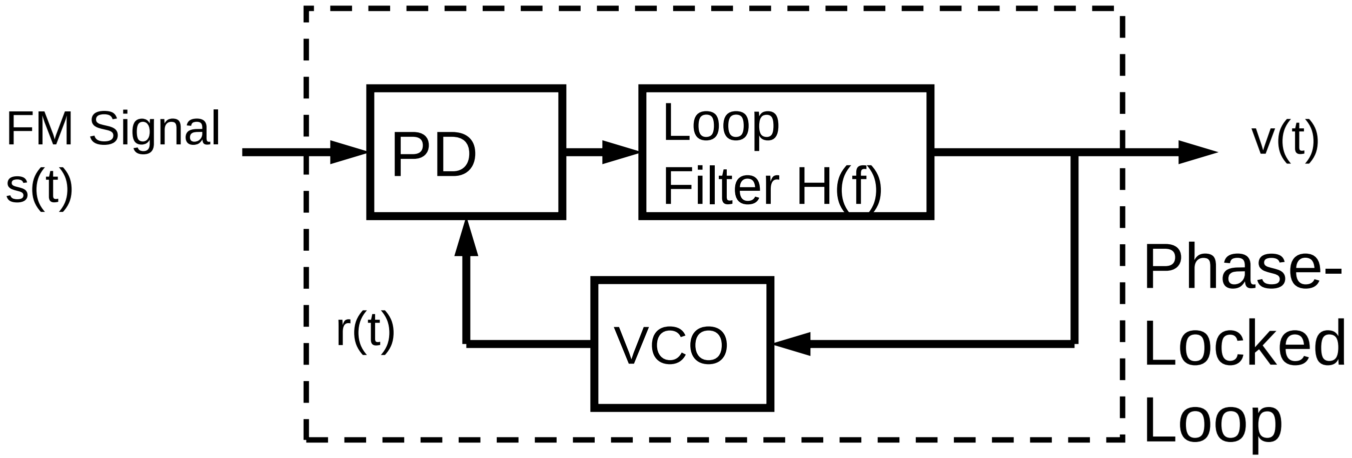
$$\boldsymbol{f_i(t) = f_c + k_f m(t)}$$

$$\boldsymbol{V_0 = V_1 - V_2}$$

The Phase Locked Loop

- Another implementation for the discriminator is the phase locked loop (PLL).
- The PLL is a negative feedback control system whose purpose is to force the frequency of the voltage controlled oscillator (VCO) to track the frequency and phase at its input.
- Has many applications in communications:
  - Carrier synchronization
  - Demodulation: e.g., DSB, FM
  - Frequency multiplication and division,
  - Frequency synthesis
  - Clock recovery circuits
- It consists of three main components:
  - Phase detector (PD)
  - Loop filter
  - Voltage controlled oscillator (VCO).

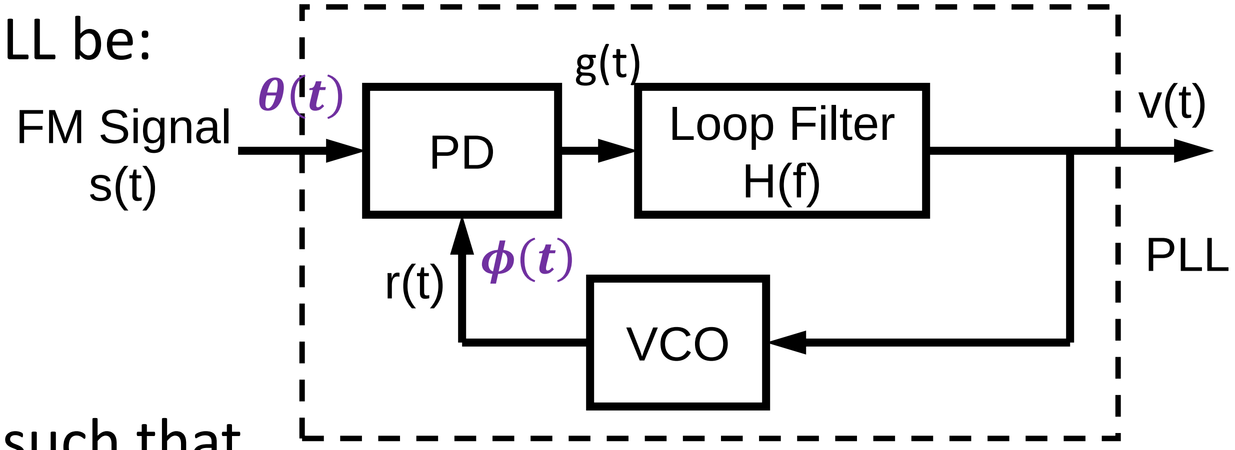
Functional Blocks of PLL



- Phase detector (PD): finds phase difference between the two inputs s(t) and r(t)
- Loop filter: provides appropriate control voltage for the voltage-controlled oscillator (VCO). It determines the order of the loop (first or second order).
- VCO: generates a signal r(t) with frequency determined by the control voltage v(t), hence the name VCO.

The Phased Locked Loop: Basic Operation

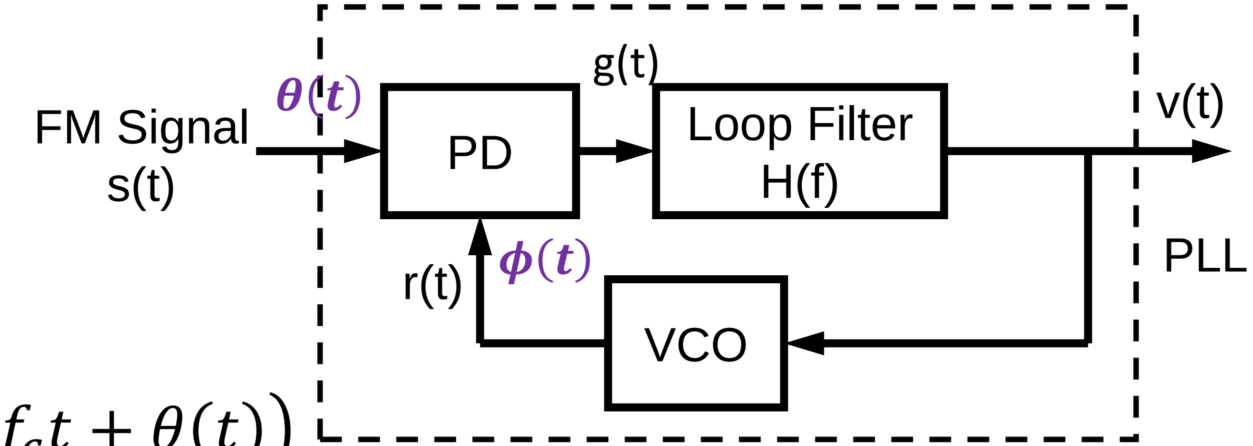
- **Initializing the loop:** Let the FM input to the PLL be:
- $s(t) = A_c \cos(2\pi f_c t + \theta(t))$;
  - $\theta(t) = 2\pi k_f \int_0^t m(t)dt$; for an FM input
- Initialize the loop by setting $\theta(t) = 0$
- The frequency of the VCO will then follow f_c; such that
  - $g(t) \cong 0$; $v(t) \cong 0$
  - $r(t) = A_c' \sin(2\pi f_c t)$
- When $\theta(t) \neq 0$ the frequency of the VCO is
- $f_r(t) = f_c + k_v v(t)$; VCO is an FM modulator
- The VCO signal will then follow
  - $r(t) = A_c' \sin(2\pi f_c t + \phi(t))$; **r(t) is an FM signal**
  - $\phi(t) = 2\pi k_v \int_0^t v(t)dt$; so that $v(t) = \left(\frac{1}{2\pi k_v}\right)\frac{d\phi(t)}{dt}$



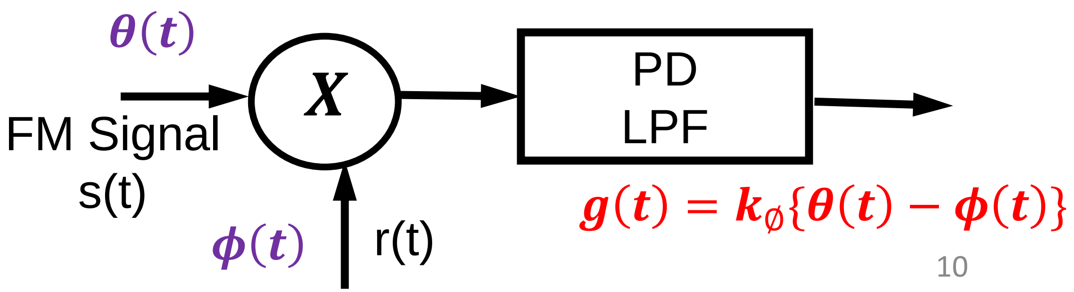
$f_r(t) = f_c + k_v v(t)$;

$f_r(t) = f_c + \frac{1}{2\pi}\frac{d\phi(t)}{dt}$;

The Phased Locked Loop: Basic Operation

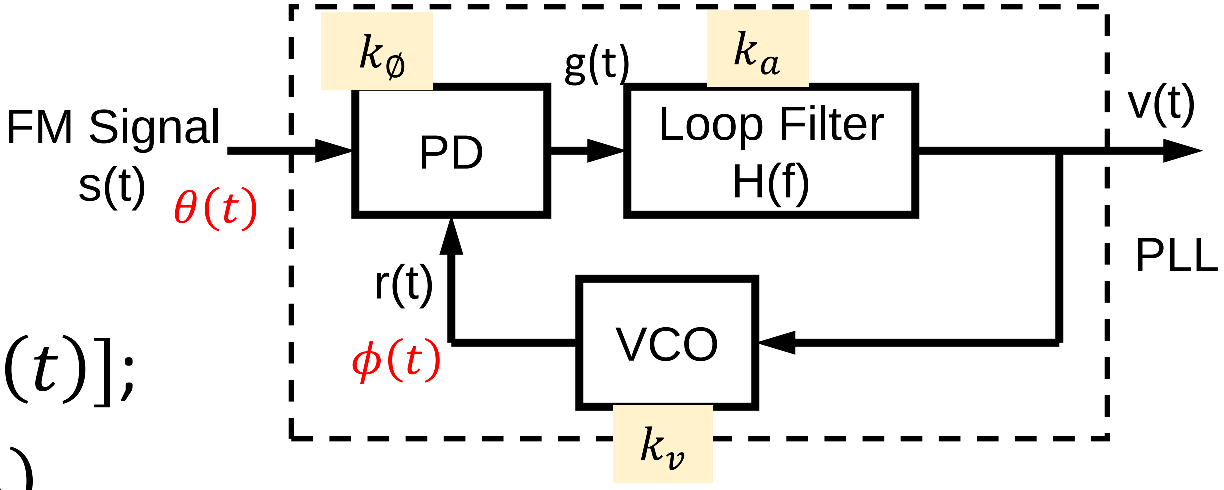


- **Phase Detector**
- Consists of a mixer followed by a LPF
- $s(t)r(t) = A_c{}'A_c \sin\big(2\pi f_c t + \phi(t)\big)\cos\big(2\pi f_c t + \theta(t)\big)$
- $= 0.5A_c{}'A_c \sin\big(4\pi f_c t + \phi(t)/2 + \theta(t)\big) + 0.5A_c{}'A_c \sin\big(\theta(t) - \phi(t)\big)$
- The output of the LPF is: $0.5A_c{}'A_c \sin\big(\theta(t) - \phi(t)\big)$
- When $\theta(t) - \phi(t)$is small, $\sin\big(\theta(t) - \phi(t)\big) \cong \theta(t) - \phi(t)$;
- $\mathbf{g(t) = k_\phi[\theta(t) - \phi(t)];}$



First Order PLL

- **Loop Output**
- In the simple case let $H(f) = k_a$
- Then, the output v(t) is:
- $v(t) = k_a g(t) = (k_a)(k_\emptyset)[\theta(t) - \phi(t)];$
- $v(t) = k[\theta(t) - \phi(t)];\ k = (k_a)(k_\emptyset)$
-



$$\theta(t) = 2\pi k_f \int_0^t m(t)dt$$

$$\phi(t) = 2\pi k_v \int_0^t v(t)dt$$

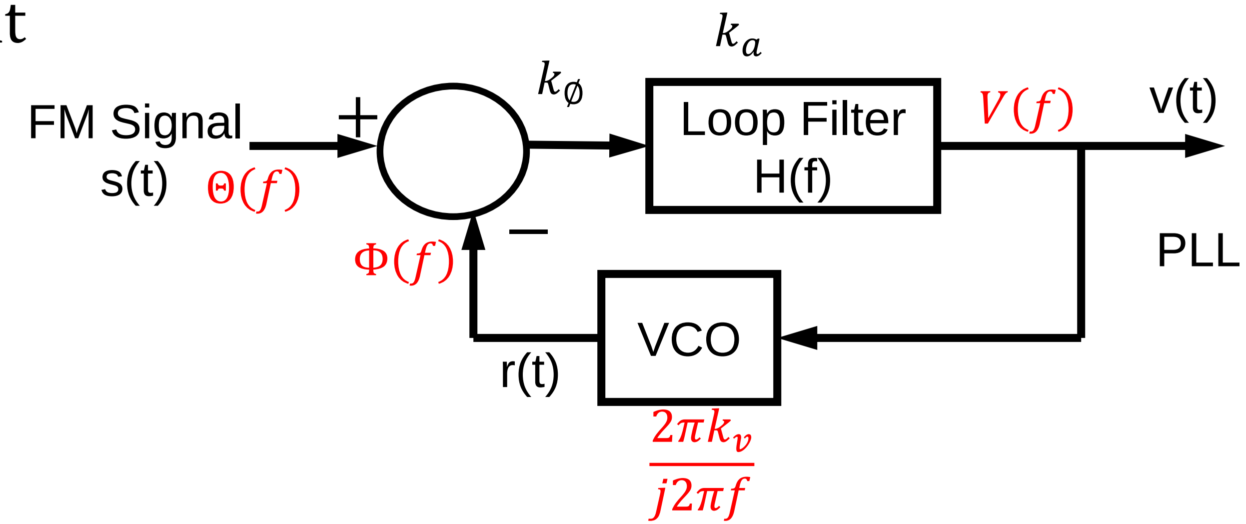
- **The two main loop equations for a first order PLL**

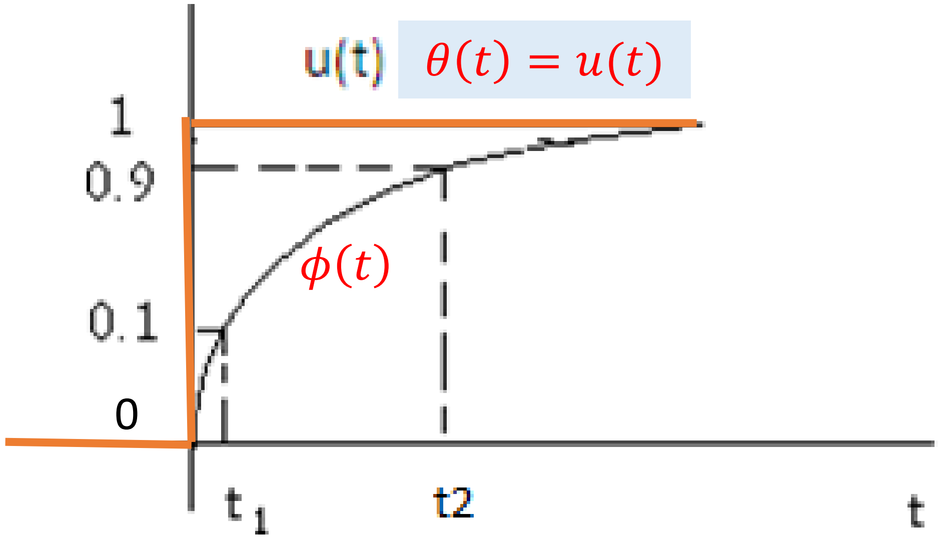
$$v(t) = k[\theta(t) - \phi(t)];$$

- $$v(t) = \left(\frac{1}{2\pi k_v}\right)\frac{d\phi(t)}{dt}$$ OR $$\phi(t) = 2\pi k_v \int_0^t v(t)dt$$

The Phased Locked Loop: Impulse and Step Responses

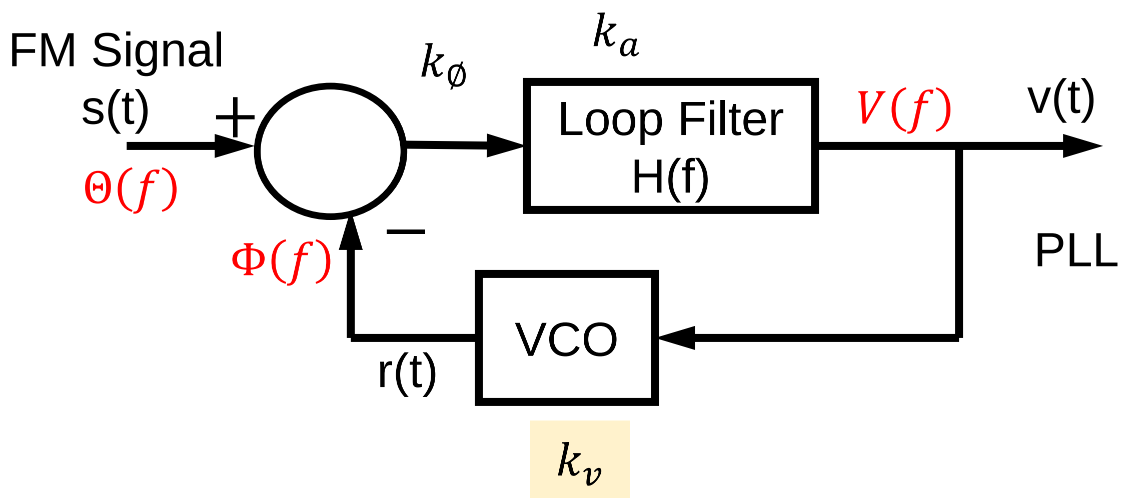
- Find loop impulse response relative to input $\boldsymbol{\theta(t)}$ **and output** $\boldsymbol{\phi(t)}$;
- $v(t) = k[\theta(t) - \phi(t)]$ (1)
- $\phi(t) = 2\pi k_v \int_0^t v(t)dt$ (2)
- Taking the Fourier transform of (1) and (2),
- $V(f) = k[\Theta(f) - \Phi(f)]$ (3)
- $\Phi(f) = \frac{2\pi k_v}{j2\pi f} V(f)$ (4)
- Combining (3), (4), the transfer function is:
- $H_1(f) = \frac{\Phi(f)}{\Theta(f))} = \frac{(2\pi)kk_v}{2\pi kk_v + j2\pi f} = \frac{K_1}{K_1 + j2\pi f}$
- $\boldsymbol{h_1(t) = K_1 e^{-K_1 t} u(t)}$; K_1: Loop Gain
- **Step Response**: If $\theta(t) = u(t)$, then
- $\boldsymbol{\phi(t) = (1 - e^{-K_1 t})u(t)}$
- Hence, as K_1 increases, $\phi(t)$ becomes closes and closer to $\theta(t)$ for any value of t.





The Phased Locked Loop: Impulse and Step Responses

- Hence, as K_1 increases, $\phi(t)$ becomes closes and closer to $\theta(t)$ for any value of t.
- For any phase input $\theta(t)$; the steady-state condition for the loop is such that:
- $\boldsymbol{\phi(t) \cong \theta(t)}$; When the loop gain is large
- $\boldsymbol{2\pi k_v \int_0^t v(t)dt \cong 2\pi k_f \int_0^t m(t)dt}$;
- locking condition
- $\Rightarrow \boldsymbol{k_v v(t) \cong k_f m(t);}$
- $\Rightarrow \boldsymbol{v(t) \cong \frac{k_f}{k_v} m(t)}$
- Hence, FM demodulation is accomplished.



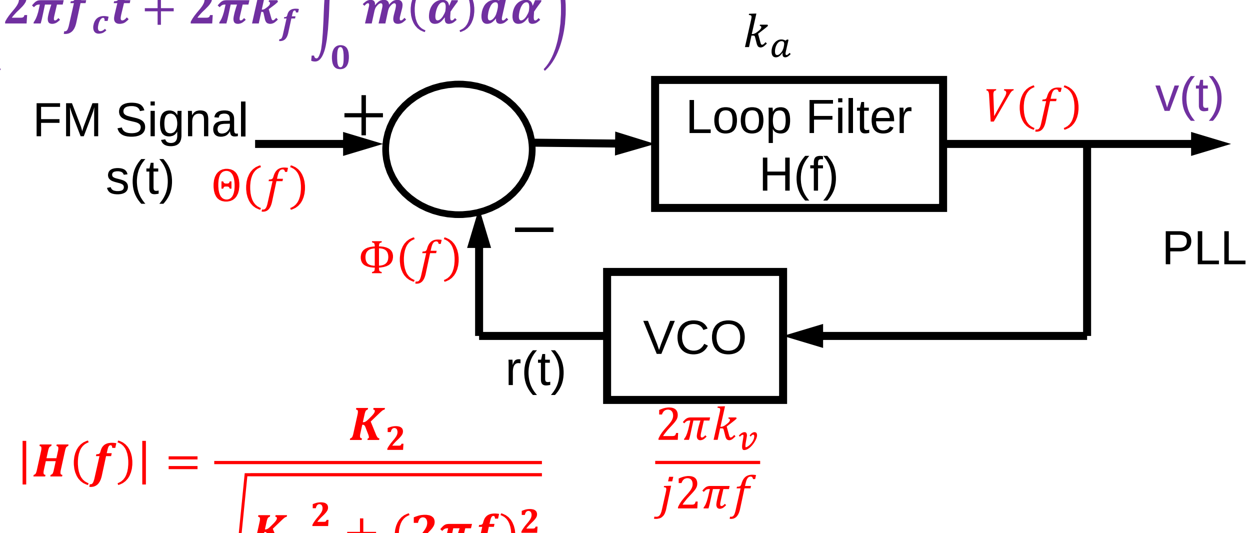
$$\mathbf{r}(\boldsymbol{t}) = \boldsymbol{A_c}' \boldsymbol{sin}\left(\boldsymbol{2\pi f_c t + 2\pi k_v \int_0^t v(t)dt}\right)$$

$$\boldsymbol{s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(t)dt\right)}$$

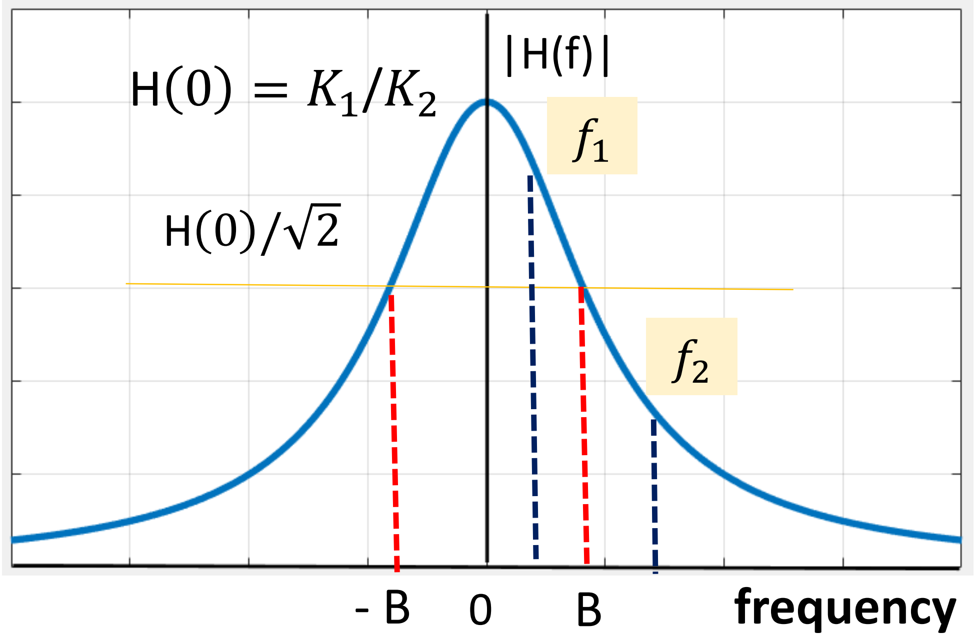
The Phased Locked Loop: Steady-State Frequency Response

- *Here,* $m(t) = A_m \cos(2\pi f t)$;
- **Find V(f)/M(f)**
- $v(t) = k[\theta(t) - \phi(t)]$ (1)
- $\phi(t) = 2\pi k_v \int_0^t v(t)dt$ (2)
- $\theta(t) = 2\pi k_f \int_0^t m(t)dt$ (3)
- Taking the Fourier transform of (1), (2), and (3)
- $V(f) = k[\Theta(f) - \Phi(f)]$ (4)
- $\Phi(f) = \frac{2\pi k_v}{j2\pi f} V(f)$ (5)
- $\Theta(f) = \frac{2\pi k_f}{j2\pi f} M(f)$ (6)
- Combining (4), (5), and (6), the transfer function is:
- $H(f) = \frac{V(f)}{M(f))} = \frac{(2\pi)kk_f}{2\pi kk_v + j2\pi f} = \frac{K_2}{K_1 + j2\pi f}$
- **PLL** acts like a LPF with 3-dB bandwidth of $B = K_1/\sqrt{2}$.
- High frequencies are attenuated more than low ones.
- $m(t) = (1)[\cos(2\pi f_1(t)] + (1)[\cos(2\pi f_2(t)]$
- $v(t) = A_1[\cos(2\pi f_1(t - t_1)] + A_2[\cos(2\pi f_2(t - t_2)]$

$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\alpha)d\alpha\right)$$



$$|H(f)| = \frac{K_2}{\sqrt{K_1{}^2 + (2\pi f)^2}}$$

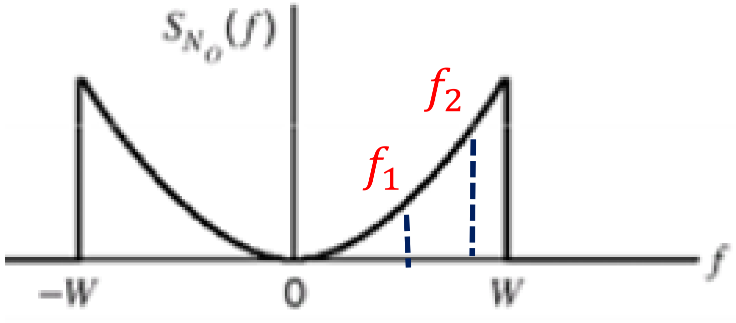


Pre-emphasis and De-emphasis in FM

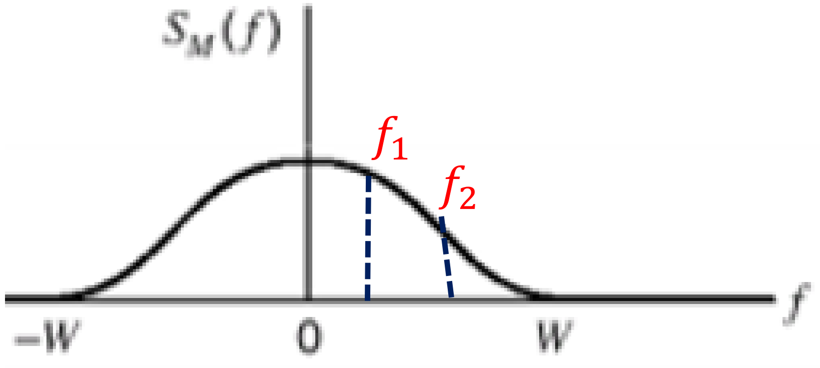
Three factors affect the quality of high frequencies at the FM demodulator output:

1. Power spectral density of message (in practice) falls off for higher frequencies.
2. Power spectral density of noise at demodulator output is proportional to the square of the frequency (will not be derived here).
3. The PLL behaves as a low pass filter, in the sense that higher frequencies will be attenuated more than low frequencies.

- These three effects severely affect the high frequencies of the signal resulting in a lower signal to noise ratio (SNR) for these frequencies. Hence, high frequencies will be distorted to a higher degree than the low frequencies.
- Pre-emphasis and de-emphasis are used to maintain an almost constant SNR over all frequencies.



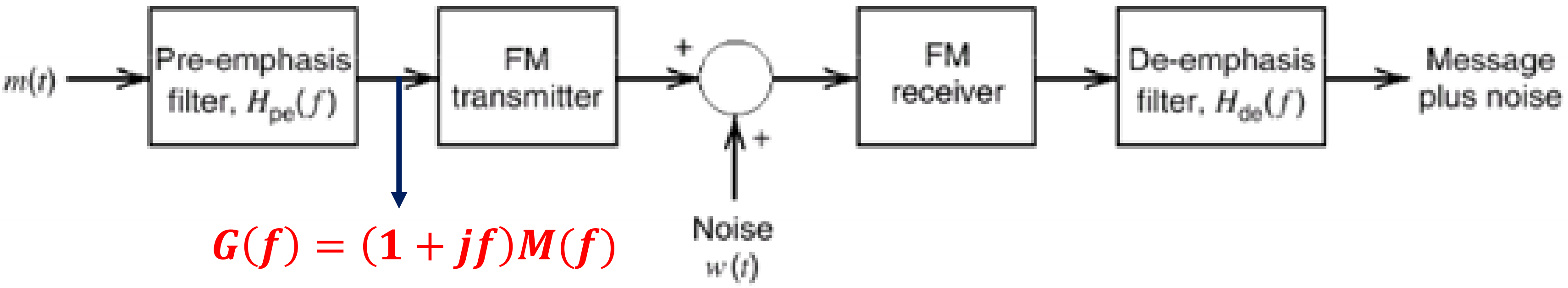
Noise PSD at PLL output



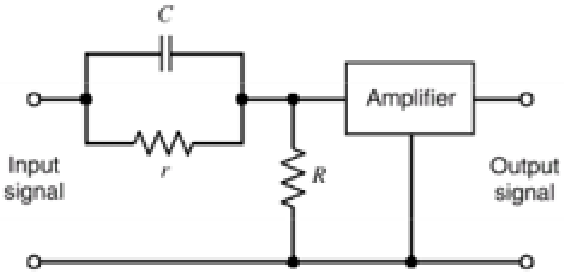
Message PSD

Pre-emphasis and De-emphasis in FM

- The pre-emphasis filter $H_{pe}(f)$ is used to emphasize the high frequency components of the message prior to modulation, and hence before noise is introduced.
- The de-emphasis filter $H_{de}(f)$ used at the receiver restores the original message signal
- $\boldsymbol{H_{pe}(f)H_{de}(f) = constant}$; for a distortion-less transmission).
- In theory, $\boldsymbol{H_{pe}(f) \propto f}$ and $\boldsymbol{H_{de}(f) \propto 1/f}$



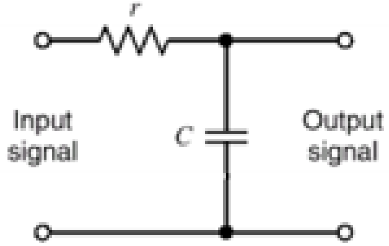
$$\boldsymbol{G(f) = (1 + jf)M(f)}$$



(a) Pre-emphasis filter

$$H_{pe}(f) \cong 1 + jf / f_0$$

$$f_0 = 1/(2\pi rC), \quad R \ll r, \quad 2\pi f rC \ll 1$$



(b) De-emphasis filter

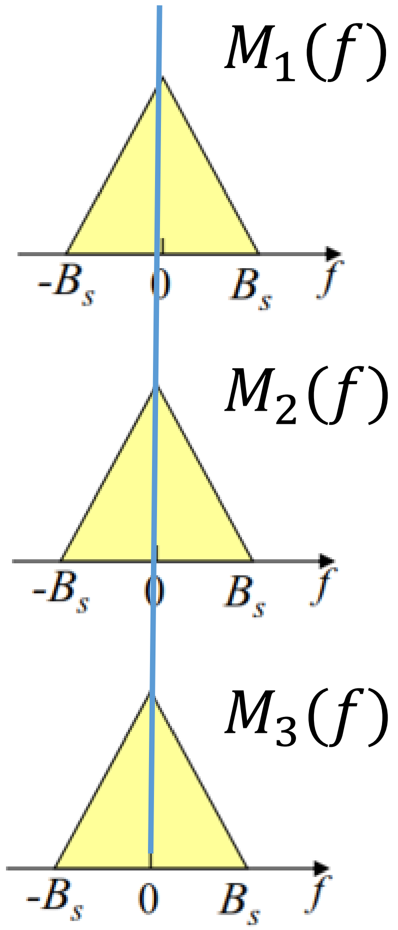
$$H_{de}(f) = \frac{1}{1 + jf / f_0}$$

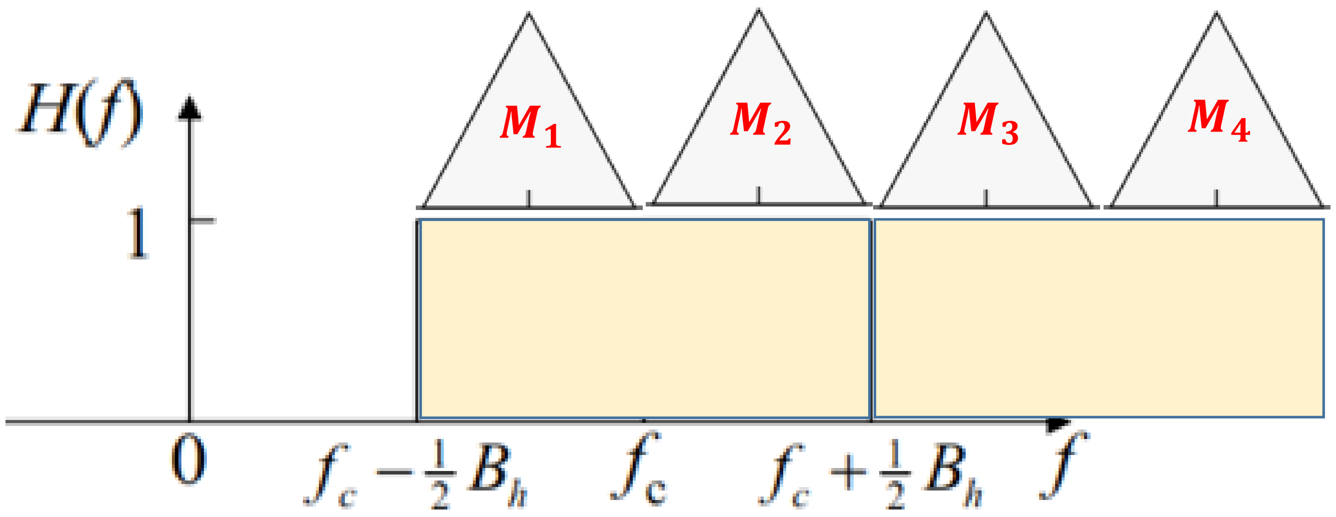
AM versus FM

- Normal AM requires simple circuits, and is very easy to generate and demodulate
- It is simple to tune, and is used in almost all short wave broadcasting.
- The area of coverage of AM is greater than FM (longer wavelengths ; lower frequencies).
- AM is power inefficient, and is susceptible to static and other forms of electrical noise.
- The main advantage of FM is its audio quality and immunity to noise. Most forms of static and electrical noise affect the amplitude, and an FM receiver will not respond significantly to such an amplitude noise.
- The audio quality of an FM signal increases as the frequency deviation increases (deviation from the center frequency), which is why FM broadcast stations use such large deviation.
- The main disadvantage of FM is the larger bandwidth it requires

Frequency Division Multiplexing

- **Multiplexing**: A technique which allows multiple users to use the same channel at the same time by assigning each user a portion of the available bandwidth without interfering with other users.
- The main two topics of this lecture are quadrature carrier modulation and frequency division multiplexing.

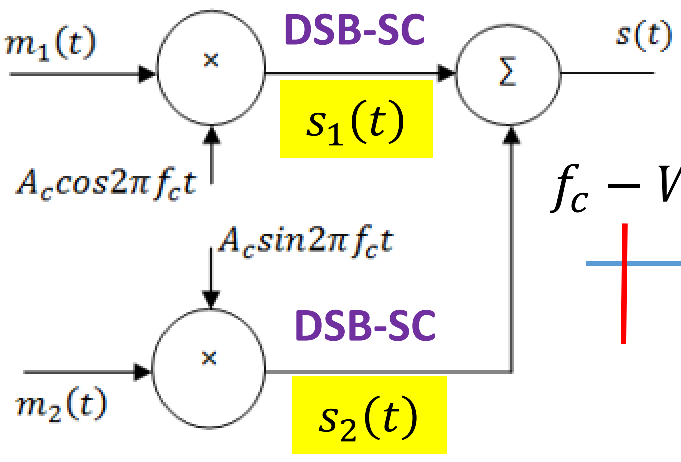




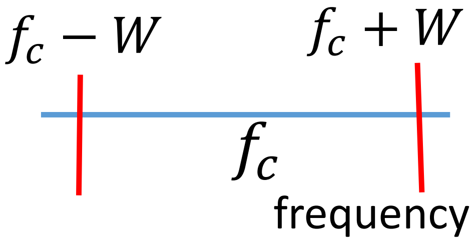
Band-pass Channel

Frequency Division Multiplexing

- **Quadrature Carrier Multiplexing:** This scheme enables two DSB-SC modulated signals to occupy the same transmission B.W and yet allows for the separation of the message signals at the receiver
- **Modulation**: $m_1(t)$ and $m_2(t)$ are low pass signals each with a B.W = W Hz .
- The composite signal is:
- $\boldsymbol{s(t) = A_c m_1(t) cos2\pi f_c t + A_c m_2(t) sin2\pi f_c t}$
- $\boldsymbol{S(f) = [A_c M_1(f - f_c) + A_c M_1(f + f_c)]/2}$
- $\boldsymbol{+ [A_c M_2(f - f_c) - A_c M_2(f + f_c)]/j2}$
- $s(t) = s_1(t) + s_2(t)$
- where $s_1(t)$ and $s_2(t)$ are both DSB-SC signals.
- B.W of $s_1(t)$ = 2W; B.W of $s_2(t)$ = 2W; B.W of $s(t)$ = 2W



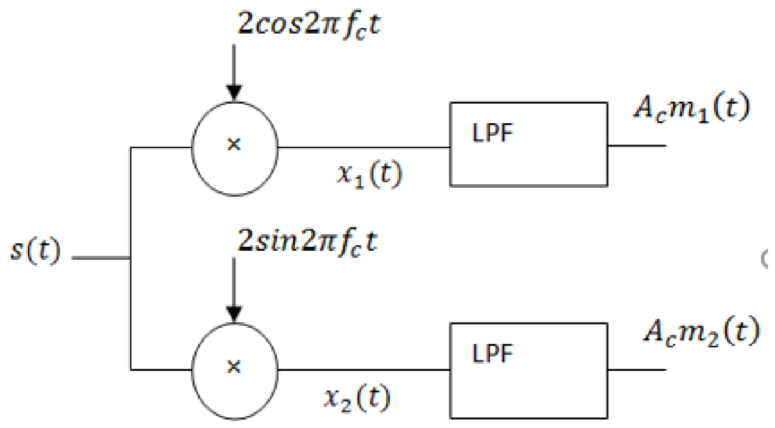
Modulation



- This method provides bandwidth conservation. That is, two DSB-SC signals are transmitted within the bandwidth of one DSB-SC signal. Therefore, this multiplexing technique provides bandwidth reduction by one half.

Frequency Division Multiplexing

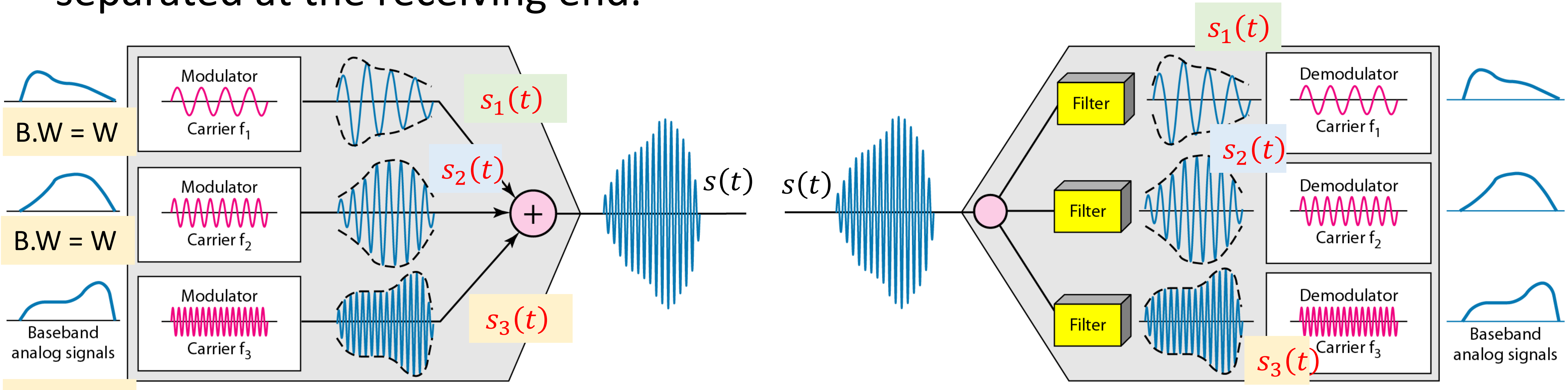
- **Quadrature Carrier Multiplexing (QAM)**
- **Demodulation:** Given $s(t)$, the objective is to recover $m_1(t)$ and $m_2(t)$ from $s(t)$. Consider first the in-phase channel
- $x_1(t) = 2cos2\pi f_c t \;\; s(t)$
  $= 2\cos 2\pi f_c t\, (A_c m_1(t)cos2\pi f_c t + \; A_c m_2(t)sin2\pi f_c t)$
- $= 2A_c m_1(t)cos^2 2\pi f_c t + 2A_c m_2(t)sin\omega_c t \; cos\omega_c t$
- $= 2A_c m_1(t)\left(\frac{1+cos2\omega_c t}{2}\right) + A_c m_2(t)sin2\omega_c t$
- $= A_c m_1(t) + \; A_c m_1(t)cos2\omega_c t + \; A_c m_2(t)sin2\omega_c t$
- After low pass filtering, the output of the in-phase channel is
- $\boldsymbol{y_1(t) = A_c m_1(t).}$
- Likewise, it can be shown that $\boldsymbol{y_2(t) = A_c m_2(t)}$.
- **Note:** Synchronization is a problem. That is to recover the message signals, it is important that the two carrier signals (the sine and the cosine functions) at the receiver should have the same phase and frequency as the signals at the transmitting side. A phase error or a frequency error will result is an interference type of distortion. That is, A component of $m_2(t)$ will appear in the in-phase channel in addition to the desired signal $m_1(t)$ and a component of $m_1(t)$ will appear at the quadrature output.



Demodulation

Frequency Division Multiplexing

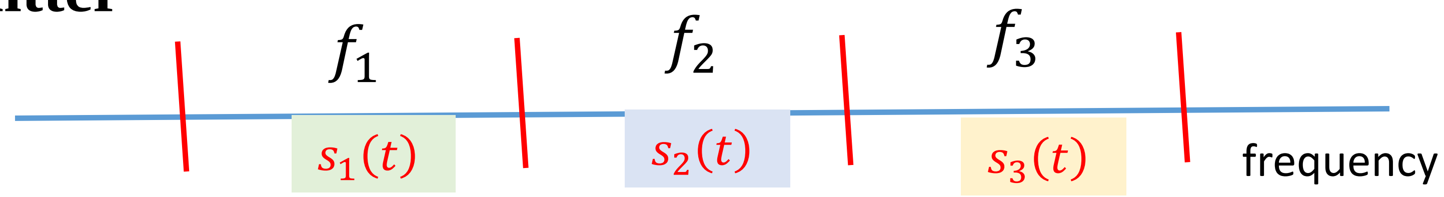
- A number of independent signals can be combined into a composite signal suitable for transmission over a common channel. The signals must be kept apart so that they do not interfere with each other and thus they can be separated at the receiving end.



$$s(t) = A_{c_1} m_1(t) \cos 2\pi f_1 t + A_{c_2} m_2(t) \cos 2\pi f_2 t + A_{c_3} m_3(t) \cos 2\pi f_3 t$$

Receiver

Transmitter



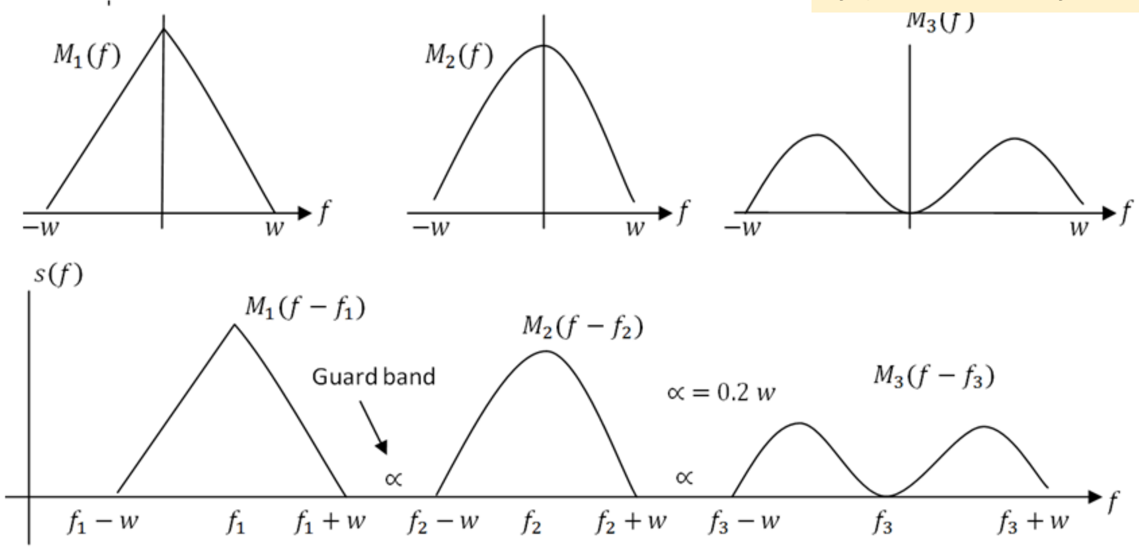
Reference: Data Communications and Networking: Forouzan

Example: Double Sideband Frequency Division Multiplexed Signals

- Let $m_1, m_2 \ and \ m_3$ be three baseband message signals each with a B.W = W.
- The composite modulated signal $s(t)$ is
- $s(t) = A_{c_1} m_1(t) cos2\pi f_1 t + A_{c_2} m_2(t) cos2\pi f_2 t + A_{c_3} m_3(t) cos2\pi f_3 t$
- $= s_1(t) + s_2(t) + s_3(t)$
- $s_1, s_2 \ and \ s_3$ are DSB-SC signals with carrier frequencies $f_1, \ f_2 \ and \ f_3$, respectively. If the spectrum of $m_1(t), \ m_2(t) \ and \ m_3(t)$ are as shown, the spectrum of $s(t)$ can be found as shown below.

$$f_2 - w \geq f_1 + w \quad or \quad f_2 - f_1 \geq 2w$$

$$f_3 - w \geq f_2 + w \quad or \quad f_3 - f_2 \geq 2w$$



Receiver